

AD-A124 698

ACTIVE CONTROL OF LINEAR PERIODIC SYSTEM WITH TWO
UNSTABLE MODES(U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGINEERING

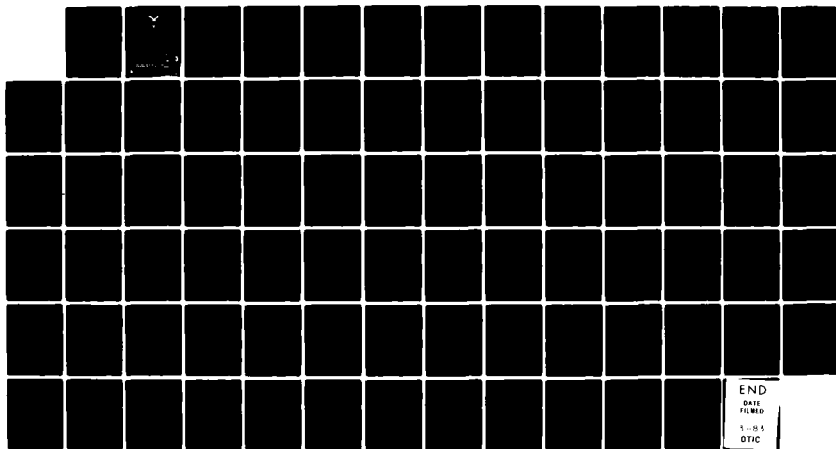
1/1

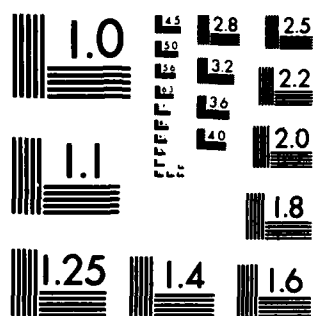
UNCLASSIFIED

G E MYERS DEC 82 AFIT/GAE/AA/82D-21

F/G 22/1

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

ADA 124698



DTIC



ACTIVE CONTROL OF LINEAR PERIODIC
SYSTEM WITH TWO UNSTABLE MODES

THESIS

by

Gregory E. Myers, B.S.E.
2nd Lt. USAF

This document has been approved
for public release and sale; its
distribution is unlimited.

DTIC
ELECTE
FEB 22 1966



AFIT/GAE/AA/82D-21

ACTIVE CONTROL OF LINEAR
PERIODIC SYSTEM
WITH TWO UNSTABLE MODES

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University (ATC)
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by
Gregory E. Myers, B.S.E.
2Lt. USAF
Graduate Aerodynamics
December 1982

Approved for public release; distribution unlimited

PREFACE

This thesis is a continuation of the work done by Yeakel in the control of linear periodic systems. While specific cases are looked at in this study, the techniques are applicable to linear periodic systems in general. This field is just beginning to be explored in depth. With the advent of computers to perform many of the tedious numerical integrations the problem becomes workable. With this thesis I hope to add one more building block to this field. More work needs to be done before we can fully understand the nature of periodic systems, and how to control them. By filling in all of the bits and pieces, eventually the field will be conquered.

Much of this thesis work was done on the computer, a tool I was not completely comfortable with. I would like to thank all the people who helped me understand the operation of the computer system, especially Dr. William Wiesel, and Captain Hugh Briggs. I would also like to thank the members of my faculty board, Captain DeWispelare, Major Wallace, and Dr. Wiesel. Most importantly I would like to thank my adviser Dr. Robert Calico for the guidance and insight he has given me this past year.

TABLE OF CONTENTS

| | |
|---|-----|
| Preface | 11 |
| List of Figures | iv |
| List of Tables | v |
| List of Notation | vi |
| Abstract | vii |
| Chapter 1 - Introduction and Background | 1 |
| Chapter 2 - Theory | 5 |
| Equations of Motion | 5 |
| Floquet Theory | 12 |
| Control Theory | 19 |
| Chapter 3 - Results | 32 |
| Parameter Space K vs α | 32 |
| Case Study - Uncontrolled | 40 |
| Case Study - Scaler Control | 51 |
| Case Study - Multiple Input Control | 59 |
| Chapter 4 - Conclusions | 65 |
| Bibliography | 68 |

LIST OF FIGURES

Figure

| | | |
|------|---|----|
| 2.1 | Elliptical Orbit Elements | 6 |
| 2.2 | 1-2-3 Rotation | 8 |
| 3.1 | Parameter Space K vs α | 39 |
| 3.2 | Uncontrolled x_1 Response | 42 |
| 3.3 | Uncontrolled x_2 Response | 43 |
| 3.4 | Uncontrolled x_3 Response | 44 |
| 3.5 | Uncontrolled x_4 Response | 45 |
| 3.6 | Uncontrolled ϕ Response | 46 |
| 3.7 | Uncontrolled First Mode Response | 47 |
| 3.8 | Uncontrolled Second Mode Response | 48 |
| 3.9 | Uncontrolled Third Mode Response | 49 |
| 3.10 | Uncontrolled Fourth Mode Response | 50 |
| 3.11 | Controlled ϕ Response for Scaler Control Case 1 | 54 |
| 3.12 | Controlled ϕ Response for Scaler Control Case 2 | 55 |
| 3.13 | Locus of Characteristic Exponents for $R=0.5$ | 57 |
| 3.14 | Locus of Characteristic Exponents for $R=-0.5$ | 57 |
| 3.15 | Locus of Characteristic Exponents for $R=-3.0$ | 57 |
| 3.16 | Locus of Characteristic Exponents for $R=-10.0$ | 58 |
| 3.17 | Locus of Characteristic Exponents for $R=-20.0$ | 58 |
| 3.18 | Locus of Characteristic Exponents for $R=-50.0$ | 58 |
| 3.19 | Controlled ϕ Response for Multiple Input Control Case 1 | 62 |
| 3.20 | Controlled ϕ Response for Multiple Input Control Case 2 | 64 |

LIST OF TABLES

Table

| | | |
|-----|--|----|
| 3.1 | Characteristic Exponents for K vs α ($e=0.5$) | 34 |
| 3.2 | Gain Function for Multiple Input Case 1 | 61 |
| 3.3 | Gain Function for Multiple Input Case 2 | 63 |

LIST OF NOTATION

| | |
|-----------|---|
| A | System Periodic Matrix |
| a | Semi-major Axis |
| B | Control Matrix |
| C | Monodromy Matrix |
| e | Orbit Eccentricity |
| f | Element of F inverse Matrix |
| F | Periodic Solution Matrix |
| I | Inertia Matrix; Moment of Inertia |
| J | Jordan Matrix; Moment of Inertia |
| K | Gain Matrix; Inertia Parameter |
| M | External Moments |
| r | Orbital Radius |
| T | Orbit Period |
| x | State Vector |
| u | Control Vector |
| α | Angular Acceleration; Spin Parameter |
| ξ | Nondimensional Distance |
| η | Modal Variable |
| Θ | Angle of Rotation |
| λ | Characteristic Exponent |
| ν | True Anomaly |
| ρ | Characteristic Multiplier |
| σ | Real Part of Complex Characteristic Exponent |
| τ | Nondimensional time |
| Φ | Principle Fundamental Matrix |
| ϕ | Angle between Spin Axis and Orbit Normal |
| Ψ | Fundamental Matrix |
| ω | Angular Velocity; Imaginary Part of Characteristic Exp. |

ABSTRACT

↘ The attitude of a spinning symmetric satellite in an elliptical orbit was analyzed. The linearized equations were formed from which the stability was determined using Floquet theory. The majority of satellite configurations examined exhibited two unstable modes. Control laws using pole placement techniques were implemented which stabilized the satellite. A scalar control provided stability but did not allow for exact pole placement of the controlled system. Multiple input control gave exact pole placement but required complicated time dependent gains. ↗

CHAPTER 1

INTRODUCTION AND BACKGROUND

Since the beginning of space flight in the late 1950's designers have been concerned with maintaining the attitude of their satellite with respect to some fixed reference. Today this is still a major concern, as the use of earth orbiting satellites is vital not only for weather and communications problems, but also for national security interests. The need to keep an unmanned satellite "looking" at a particular point on the earth's surface, whether it be a storm front or a foreign military installation, is an important problem facing satellite designers. Various methods have been used in the past to achieve this "line of sight" precision, involving both passive and active control mechanisms. For satellites in nearly circular orbits a gravity gradient stabilization has been used. This method has many restrictions on satellite design and also requires nearly circular orbits. Gravity gradient stabilization yields very low natural frequencies, and hence any disturbances take a long time to die out; not a good condition for precision surveillance. More active attitude control devices are used such as magnetic torquers, mass movement and momentum exchange devices, and jet thrusters. Each has its own advantages and disadvantages depending on the mission requirements. The most important factor in selecting a control device, and for the entire satellite package, is weight. The lighter the overall

system is, the higher orbit it can be put into, and the longer it will stay in space. Therefore the lighter the control system the more weight is available for payload. The more payload implies a higher degree of mission accomplishment or sophistication.

One active control device long employed is the gyroscope. A gyroscope, having a large amount of angular momentum with torque free motion, results in a nearly constant angular momentum vector. Thus it is ideal for an attitude reference device. Gyros have long been used for aircraft navigation and so their use in spacecraft seemed natural. To simplify the problem the idea became to make the entire satellite gyroscopic by spinning it about some axis. As early as 1963 Kane and Shippey investigated the stability of a spinning satellite in a circular orbit, (5;111). This yielded equations which were nonlinear and nonautonomous. Later in 1966 Kane and Barba investigated the problem of a symmetric satellite in an elliptical orbit, spinning about its axis of inertial symmetry, (4;402). Again the system was nonlinear and non-autonomous.

In both cases the equations were linearized about an equilibrium point giving linear equations with periodic coefficients whose stability was checked using Floquet theory. In the case of a satellite in an elliptical orbit the stability was found to be dependent on the inertia properties, the orbit eccentricity, and the spin rate of the satellite.

As the need for more precise line of sight capability has increased, the use of gravity gradient stabilization and other schemes overly restrictive of satellite design, has become undesirable. Some sort of on board control system, or autopilot is needed. For the case of the symmetric spinning satellite in elliptical orbit, many of the satellite configurations are inherently unstable. As the eccentricity is increased more of the cases become unstable. A control scheme needs to be implemented so that these unstable configurations can be made stable and thus operational. The linearized system of equations describing this case (Kane and Barba) have periodic coefficients and so classical control techniques used for systems with constant coefficients are not directly applicable and must be modified. Very little work has been done on control of periodic systems. Shelton developed a control scheme for periodic systems, in his thesis, for a satellite in orbit about the earth moon Lagrange point L4. (Ref;9) Yeakel used the same principles in developing a control scheme for the case of an unsymmetric satellite in a circular orbit, and a symmetric satellite in an elliptical orbit. (Ref;11)

This control scheme involves transforming the state variables to modal variables so the designer can look at the individual modes and their respective stability. Control terms can then be applied to the individual modes. The control terms are based on feedback of the selected modes, modified by selected gains. In Shelton's and Yeakel's work the controller was

designed to stabilize a system where one mode was unstable. Many of the cases presented in this study have two unstable modes. This makes the control a more complicated process due to the coupling of the unstable modes. This results in unpredictable results using the scalar control developed by Shelton and Yeakel. The use of multiple input control for a system with two unstable modes is therefore developed. This control scheme gives predictable results, but has its own special drawbacks, namely complicated gains.

This study addresses the case of a symmetric satellite in an elliptical orbit, spinning about its axis of inertial symmetry, where two modes are unstable. The stability is dependent on the eccentricity, inertia properties, and the spin rate of the satellite, and the majority of cases involve two unstable modes. Thus the ability to adequately control such a case is extremely important. It is the purpose of this study to investigate both scalar control schemes and multiple input control schemes in stabilizing such a system with two unstable modes.

CHAPTER 2

THEORY

In this chapter the equations of motion of an inertially symmetric satellite in an elliptical orbit about the earth are developed. The satellite is spinning about its axis of inertial symmetry. Also the equations describing the attitude motion of the satellite are presented. These attitude equations will be linearized about an equilibrium point to describe the system in a conventional linear form. The system is time periodic due to the nature of the orbital motion, therefore Floquet theory can be used to make a statement about the stability of the attitude motion. An unstable system can be made stable by inserting a feedback control term in the model. This control term can either be scalar control or multiple input control. Both cases are developed.

EQUATIONS OF MOTION

The development that follows is taken from Kane and Barba (4;402). Two basic equations describe the trajectory of a satellite in orbit about a spherically symmetric attracting body. These equations are developed in any introductory astrodynamics text (10;31). The equations are:

$$\ddot{r} - r\dot{\nu}^2 + \frac{n^2 a^3}{r^2} = 0 \quad (2.1)$$

and,

$$r^2 \ddot{\nu} = a^2 n \sqrt{1 - e^2} \quad (2.2)$$

where

$$n = 2\pi/T \quad (2.3)$$

and where T is the period of the orbit. The orbital elements; r, ν, a , and e are shown in Figure 2.1;

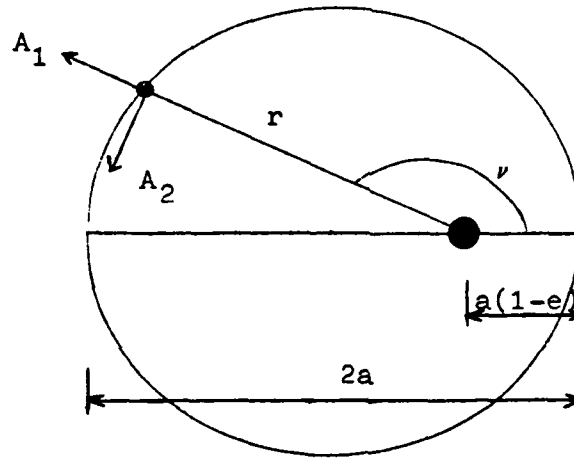


Figure 2.1 Elliptical Orbit Elements

The variables may be nondimensionalized by changing to a new system such that:

$$\tau = nt \quad \zeta = \frac{r}{a} \quad (2.4)$$

By rearranging equation (2.2),

$$\ddot{\nu} = \frac{a^2 n \sqrt{1 - e^2}}{r^2} \quad (2.5)$$

substituting this into equation (2.1) gives;

$$\ddot{\zeta} - \frac{a^4 n^2 (1 - e^2)}{\zeta^3} + \frac{n^2 a^3}{\zeta^2} = 0 \quad (2.6)$$

Now using the nondimensional variables and denoting differentiation with respect to τ by primes:

$$\zeta'' + \frac{(e^2 - 1)}{\zeta^3} + \frac{1}{\zeta^2} = 0 \quad (2.7)$$

The initial conditions for this differential equation are found by assuming the satellite starts at perigee, therefore;

$$\begin{aligned} r_p &= a(1 - e) & \dot{r}_p &= 0 \\ \text{or,} & & & \\ \zeta(0) &= 1 - e & \zeta'(0) &= 0 \end{aligned} \quad (2.8)$$

Nondimensionalizing equation (2.5) gives;

$$\nu' = \frac{\sqrt{1 - e^2}}{\zeta^2} \quad (2.9)$$

and,

$$\nu'' = \frac{-2\sqrt{1 - e^2}\zeta'}{\zeta^3} \quad (2.10)$$

It is worth noting that ζ is a periodic function of τ .

Now the equations to describe the attitude motion of the satellite can be developed. First an orbital reference frame A is defined. Frame A is such that A_1 points outward along the orbit radius r , A_2 is perpendicular to A_1 in the orbital plane such that at perigee the satellite is moving only in the positive A_2 direction, and A_3 is perpendicular to both A_1 and A_2 , pointing out of the orbital plane. A body axis frame X is obtained by means of a 1-2-3 rotation through

the angles θ_1 , θ_2 , and θ_3 . Since the satellite is assumed to be inertially symmetric a nodal axis system C is introduced which is obtained from A by means of a 1-2 rotation through θ_1 and θ_2 . See Figure 2.2.

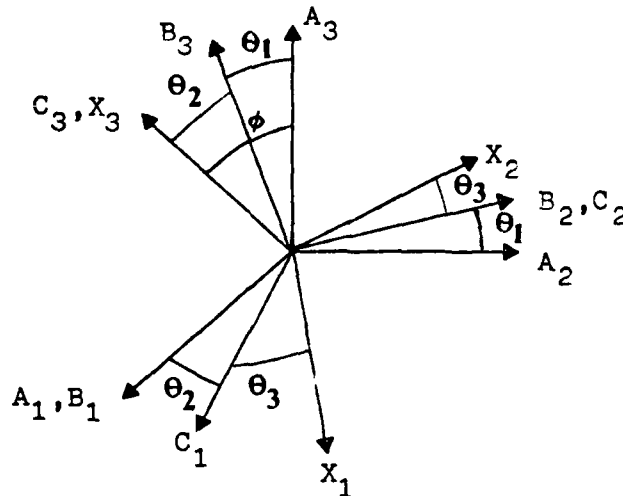


Figure 2.2 1-2-3 Rotation

Expressing the inertial angular velocity components in the C frame, including the orbital rate $\dot{\nu}$ A_3 ;

$$\begin{aligned}\omega_1 &= \dot{\theta}_1 \cos \theta_2 - \dot{\nu} \cos \theta_1 \sin \theta_2 \\ \omega_2 &= \dot{\theta}_2 + \dot{\nu} \sin \theta_1 \\ \omega_3 &= \dot{\theta}_1 \sin \theta_2 + \dot{\theta}_3 + \dot{\nu} \cos \theta_1 \cos \theta_2\end{aligned}\tag{2.11}$$

The moment equation (7;468) gives;

$$\vec{M} = \frac{d^C}{dt} \left[\mathbf{I} \right]_C \left[\vec{\omega}^1 \right]_C + \left[\vec{\omega}^1 \right]_C \times \left[\mathbf{I} \right]_C \left[\vec{\omega}^1 \right]_C \tag{2.12}$$

with the applied moments being (4;403);

$$\begin{aligned}
M_1 &= 0 \\
M_2 &= 3\frac{n^2}{f^3}(I - J)\sin\theta_2\cos\theta_2 \\
M_3 &= 0
\end{aligned}
\tag{2.13}$$

Now from Euler's equations;

$$\begin{aligned}
\alpha_1 + \left(\frac{J - I}{I} \right) \omega_2 \omega_3 &= \frac{M_1}{I} \\
\alpha_2 + \left(\frac{I - J}{I} \right) \omega_1 \omega_3 &= \frac{M_2}{I} \\
\alpha_3 + \left(\frac{I - I}{J} \right) \omega_1 \omega_2 &= \frac{M_3}{J}
\end{aligned}
\tag{2.14}$$

where the α 's are the angular accelerations and the I's and J's are defined from the mass moment of inertia matrix;

$$[I]_C = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & J \end{bmatrix}
\tag{2.15}$$

for a symmetric satellite. Defining an inertia parameter K such that;

$$K = \frac{J - I}{I}
\tag{2.16}$$

The inertia properties can be fully described in this one parameter. The acceleration terms α_i are found using the equation;

$$\bar{\alpha} = \frac{d\bar{\omega}^{-1}}{dt} = \frac{d}{dt} C\bar{\omega}^{-1} + C\bar{\omega}^{-1} \times B\bar{\omega}^{-1}
\tag{2.17}$$

which given in component form are;

$$\begin{aligned}
\alpha_1 &= -\ddot{\nu} \cos \theta_1 \sin \theta_2 + \dot{\nu} (\dot{\theta}_1 \sin \theta_1 \sin \theta_2 - \dot{\theta}_2 \cos \theta_1 \cos \theta_2 + \dot{\theta}_3 \sin \theta_1) \\
&\quad + \ddot{\theta}_1 \cos \theta_2 - \dot{\theta}_1 \dot{\theta}_2 \sin \theta_2 + \dot{\theta}_2 \dot{\theta}_3 \\
\alpha_2 &= \ddot{\nu} \sin \theta_1 + \dot{\nu} (\dot{\theta}_1 \cos \theta_1 + \dot{\theta}_1 \cos \theta_1 \sin \theta_2) + \ddot{\theta}_2 - \dot{\theta}_3 \dot{\theta}_1 \cos \theta_2 \\
\alpha_3 &= \ddot{\nu} \cos \theta_1 \cos \theta_2 - \dot{\nu} \dot{\theta}_1 \sin \theta_1 \cos \theta_2 - \dot{\nu} \dot{\theta}_2 \cos \theta_1 \sin \theta_2 + \\
&\quad - \ddot{\theta}_1 \sin \theta_2 + \dot{\theta}_1 \dot{\theta}_2 \cos \theta_2 + \ddot{\theta}_3
\end{aligned} \tag{2.18}$$

Now having solved for the α 's, equation (2.14) becomes;

$$\begin{aligned}
\alpha_1 + K\omega_1\omega_3 &= 0 \\
\alpha_2 - K\omega_1\omega_3 &= -3n^2 K \sin \theta_2 \cos \theta_2 / \zeta^3 \\
\alpha_3 &= 0
\end{aligned} \tag{2.19}$$

Consider the motion such that $\theta_1 = \theta_2 = 0$ and $\theta_3 = -\nu + \omega_3 t$ where $\omega_3 = n(1 + \alpha)$ with α a constant representing the spin rate about the C_3 axis. Under these conditions the accelerations $\alpha_1 = 0$ and the external moments $M_1 = 0$. This represents an equilibrium point with the C_3 axis remaining perpendicular to the orbital plane. Now if the satellite is perturbed from this equilibrium state, there is no guarantee the motion will return the satellite to this orientation. To determine the stability of the satellite, state variables are defined such that;

$$x_1 = \theta_1 \quad x_2 = \theta_2 \quad x_3 = \dot{\theta}_1 \quad x_4 = \dot{\theta}_2 \tag{2.20}$$

Linearizing about the equilibrium point described above, using

equations 2.11, 2.18, and 2.19, gives the perturbation equations in state vector form;

$$\bar{x}' = [A] \bar{x} \quad (2.21)$$

where

$$\bar{x}^T = [\theta_1, \theta_2, \theta_1', \theta_2']$$

hence

$$\begin{Bmatrix} \theta_1' \\ \theta_2' \\ \theta_1'' \\ \theta_2'' \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ A_{31} & \nu'' & 0 & A_{34} \\ -\nu'' & A_{42} & -A_{34} & 0 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_1' \\ \theta_2' \end{Bmatrix} \quad (2.22)$$

and

$$\begin{aligned} A_{11} &= A_{12} = A_{14} = A_{21} = A_{22} = A_{23} = A_{33} = A_{44} = 0 \\ A_{13} &= A_{24} = 1 \\ A_{31} &= -\nu'(\theta_3' + K\nu' + K\theta_3') \\ A_{32} &= -A_{41} = \nu'' \\ A_{34} &= \nu' - \theta_3' - K(\nu' + \theta_3') = -A_{43} \\ A_{42} &= A_{31} - 3K/\zeta^3 \end{aligned} \quad (2.23)$$

This expresses the A matrix in terms of the variables ν' , ν'' , θ_3' and ζ . The variables ν' and ν'' can be eliminated from the matrix by noting equations (2.9) and (2.10). θ_3 can be eliminated by noting the third of equation (2.14); $\alpha_3 = 0$ which when linearized is;

$$\nu' + \theta_3' = \text{constant} \quad (2.24)$$

let that constant be $(\alpha + 1)$, then;

$$\theta_3' = (1 + \alpha) - \frac{\sqrt{1 - e^2}}{\zeta^2} \quad (2.25)$$

Making these substitutions in the A matrix, the elements are only functions of the variables ζ and its derivative ζ' .

Additionally the elements depend upon K , α , and e , specifically;

$$A_{11} = A_{12} = A_{14} = A_{21} = A_{22} = A_{23} = A_{33} = A_{44} = 0$$

$$A_{13} = A_{24} = 1$$

$$A_{31} = (1 - e^2)\zeta^{-4} - (1 + \alpha)(1 + K)(1 - e^2)^{\frac{1}{2}}\zeta^{-2}$$

$$A_{32} = -2(1 - e^2)\zeta^{-3}\zeta' = -A_{41} \quad (2.26)$$

$$A_{34} = 2(1 - e^2)^{\frac{1}{2}}\zeta^{-2} - (1 + \alpha)(1 + K) = -A_{43}$$

$$A_{42} = A_{31} - 3K\zeta^{-3}$$

FLOQUET THEORY

It is important to note the A matrix is periodic with period T, that is;

$$A(\tau) = A(\tau + T) \quad (2.27)$$

where T is the period of one orbit. The stability of the motion can now be analyzed using Floquet theory. Floquet theory is used because the matrix A is a time periodic

function and does not lend itself to the same stability analysis as that of a constant A matrix. The fundamental matrix for this system must satisfy the matrix differential equation;

$$\Phi'(\tau) = A(\tau)\Phi(\tau) \quad (2.28)$$

This Φ is one fundamental matrix of the system. Floquet theory states that the $\Phi(\tau)$ may be written in the form;

$$\Phi(\tau) = P(\tau) e^{\Gamma \tau} \quad (2.29)$$

where $P(\tau)$ is a periodic matrix with the same period as the original system, and the matrix Γ is a constant matrix. It is desirable to have the Γ matrix in diagonal form, (i.e. Jordan form). This may be done by means of a similarity transformation and a new fundamental matrix ψ is therefore given by;

$$\psi(\tau) = F(\tau) e^{J \tau} \quad (2.30)$$

$F(\tau)$ is periodic such that,

$$F(\tau) = F(\tau + T) \quad (2.31)$$

and J is the Jordan form of Γ .

Now returning to the form of equation (2.29) to write;

$$\Phi(\tau + T) = P(\tau + T) e^{\Gamma(\tau + T)} \quad (2.32)$$

but

$$P(\tau) = P(\tau + T) \quad (2.33)$$

therefore,

$$\Phi(\tau + T) = P(\tau) e^{\Gamma \tau} e^{\Gamma T} = \Phi(\tau) e^{\Gamma T} \quad (2.34)$$

or

$$\Phi(\tau + T) = \Phi(\tau) C \quad (2.35)$$

Here C is a constant matrix called the monodromy matrix. The eigenvalues of the Γ matrix are called the characteristic exponents, λ_i . The eigenvalues of C are called the characteristic multipliers ρ_i . By letting $\tau = 0$ the monodromy matrix can readily be found;

$$\Phi^{-1}(0)\Phi(T) = C \quad (2.36)$$

If Φ is the principle fundamental matrix;

$$\Phi(0) = I \quad (2.37)$$

Φ can be solved numerically by recalling equation (2.28) and given the initial conditions (2.37).

The eigenvalues of the monodromy matrix are related to the characteristic exponents by;

$$\lambda_i = \frac{1}{T} \ln \rho_i \quad (2.38)$$

The stability of the system is determined by the characteristic exponents. If all of the characteristic exponents are

less than zero the system is stable. If just one of the characteristic exponents is greater than zero the system is unstable. For the case where the characteristic multipliers are complex, the relationship between them and the characteristic exponents is;

$$\lambda_i = \frac{1}{T}(\ln r_i + j \arg \rho_i) \quad (2.39)$$

where

$$\begin{aligned} r_i &= \sqrt{\operatorname{Re}(\rho_i)^2 + \operatorname{Im}(\rho_i)^2} \\ \arg \rho_i &= \arctan(\operatorname{Im}(\rho_i)/\operatorname{Re}(\rho_i)) \end{aligned} \quad (2.40)$$

In this case all real parts of the characteristic exponents must be nonpositive for stability. A positive real part means the system is unstable.

The system stability can be determined from the monodromy matrix, but the periodic solution matrix will be needed for implementation of control. A solution to the original system can be written as (3;234);

$$\bar{x}(\tau) = \psi(\tau)\psi^{-1}(0)\bar{x}(0) \quad (2.41)$$

where ψ is any fundamental matrix and $\bar{x}(0)$ is the initial condition vector, assumed given. Using the fundamental matrix of equation (2.30), and taking its inverse gives;

$$\psi^{-1}(\tau) = e^{-J\tau} F^{-1}(\tau) \quad (2.42)$$

where at $\tau = 0$,

$$\psi^{-1}(0) = e^0 F^{-1}(0) = F^{-1}(0) \quad (2.43)$$

using this in equation (2.41);

$$\bar{x}(\tau) = F(\tau) e^{J\tau} F^{-1}(0) \bar{x}(0) \quad (2.44)$$

It is important to note that $F(\tau + T) = F(\tau)$. In terms of the principle fundamental matrix $\bar{x}(\tau)$ may be expressed as;

$$\bar{x}(\tau) = \Phi(\tau) \Phi^{-1}(0) \bar{x}(0) \quad (2.45)$$

Evaluating at $\tau = T$,

$$\bar{x}(T) = \Phi(T) \Phi^{-1}(0) \bar{x}(0) \quad (2.46)$$

where from equation (2.36),

$$\Phi(T) \Phi^{-1}(0) = \Phi(T) I = \Phi(T) = C \quad (2.47)$$

where C is the monodromy matrix. Evaluating equation (2.44) at $\tau = T$ and then equating to equation (2.46) gives;

$$F(T) e^{JT} F^{-1}(0) \bar{x}(0) = C \bar{x}(0) \quad (2.48)$$

or by rearranging,

$$F(T) e^{JT} - C F(0) = \bar{0} \quad (2.49)$$

Since $F(\tau)$ is periodic, $F(0) = F(T)$ so equation (2.49) becomes;

$$F(0) e^{JT} - C F(0) = \bar{0} \quad (2.50)$$

The matrix e^{JT} is diagonal and hence the individual columns of the $F(0)$ matrix can be written as;

$$\rho_i \bar{F}_i(0) - C \bar{F}_i(0) = \bar{0} \quad (2.51)$$

with ρ_i being the i 'th diagonal element of e^{JT} . Equation (2.51) may be recognized as a basic eigenvalue problem.

Written in a more familiar form;

$$[\rho I - C] \bar{F}(0) = \bar{0} \quad (2.52)$$

Hence the $F(0)$ matrix is a matrix whose columns are the eigenvectors of the monodromy matrix C .

Referring to equation (2.30) again, taking the derivative of ψ with respect to τ gives;

$$\psi'(\tau) = F'(\tau) e^{J\tau} + F(\tau) J e^{J\tau} \quad (2.53)$$

Since ψ is a fundamental matrix it must satisfy equation (2.29), i.e.

$$\psi'(\tau) = A(\tau) \psi(\tau) \quad (2.54)$$

Using the expression for ψ and ψ' in equation (2.54) gives,

$$F'(\tau) e^{J\tau} + F(\tau) J e^{J\tau} = A(\tau) F(\tau) e^{J\tau} \quad (2.55)$$

By cancelling out the $e^{J\tau}$ terms and rearranging;

$$F' = AF - FJ \quad (2.56)$$

This equation along with the initial condition matrix found in equation (2.52) can be solved numerically for $F(\tau)$.

Summarizing the steps to determine the stability of the system;

$$\bar{x}' = A(\tau)\bar{x}$$

are as follows;

1) Determine the principle fundamental matrix Φ by numerically integrating equation (2.28) with initial conditions

$$\Phi(0) = I \quad (2.37)$$

$$\Phi'(\tau) = A(\tau)\Phi(\tau) \quad (2.28)$$

2) Determine the monodromy matrix C from equation (2.36)

$$\Phi^{-1}(0)\Phi(T) = C \quad (2.36)$$

3) Find the eigenvalues of the monodromy matrix C , ρ_i , and therefore the characteristic exponents λ_i by;

$$\lambda_i = \frac{1}{T} \ln \rho_i \quad (2.38)$$

If all the λ_i have negative real parts the system is stable, otherwise it is unstable.

Summarizing the steps to determine the periodic F matrix of equation (2.30) are as follows;

1) Find eigenvectors of the monodromy matrix from equation (2.52). These form the initial conditions for the F matrix.

$$\left[\rho I - C \right] \bar{f}(0) = \bar{0} \quad (2.52)$$

2) Solve equation (2.56) for F by numerically integrating.

$$F' = AF - FJ \quad (2.56)$$

Now all elements of the general solution form of equation (2.30) are known.

$$\psi(\tau) = F(\tau) e^{J\tau} \quad (2.30)$$

where J is the Jordan matrix, (7;267).

Equations (2.28) and (2.56) can be solved numerically, given the initial conditions. The elements of the F matrix are needed as functions of τ and since they are periodic, may be stored as Fourier coefficients.

CONTROL THEORY

The modal control of linear constant coefficient systems is well understood. The application of modal control for linear periodic systems is however not well developed and is the subject of this section. Consider a change of state variables from x to a new modal variable η , given by;

$$\bar{x}(\tau) = F(\tau) \bar{\eta}(\tau) \quad (2.57)$$

where $F(\tau)$ is the same matrix as in equation (2.30). Taking the derivative of x with respect to τ gives;

$$\bar{x}' = F' \bar{\eta} + F \bar{\eta}' \quad (2.58)$$

Substituting equation (2.57) and (2.58) into the original system of equations (2.21);

$$F' \bar{\eta} + F \bar{\eta}' = A F \bar{\eta} \quad (2.59)$$

By rearranging,

$$\bar{\eta}' = F^{-1} A F \bar{\eta} - F^{-1} F' \bar{\eta} \quad (2.60)$$

Using equation (2.56) to change equation (2.60) to;

$$\bar{\eta}' = \left[F^{-1} A F - F^{-1} (A F - F J) \right] \bar{\eta} \quad (2.61)$$

or,

$$\bar{\eta}' = \left[F^{-1} A F - F^{-1} A F + F^{-1} F J \right] \bar{\eta} \quad (2.62)$$

and finally,

$$\bar{\eta}' = J \bar{\eta} \quad (2.63)$$

This represents a much easier system to deal with since J is a constant matrix. If all the characteristic exponents are real and distinct, J is set to have elements only on the main diagonal. If this is the case, the system described by equation (2.63) reduces to n uncoupled ordinary differential equations, hence the new η variables are called the modal variables. Each η_i represents a different mode. The stabil-

ity of one mode is independent of the other modes. In the state variables each mode η_1 participates in the overall motion of the system. If the characteristic exponents are complex, J is put in block diagonal form, with the imaginary parts on the off diagonal. In this case two of the modes become coupled and the differential equations become more involved.

The feedback control term added to the basic system, changes the mathematical model to look like;

$$\bar{x}' = A\bar{x} + B\bar{u} \quad (2.64)$$

where B is a feedback application matrix and \bar{u} is the control vector. Expressing this equation in modal variables yields;

$$\bar{\eta}' = J\bar{\eta} + F^{-1}B\bar{u} \quad (2.65)$$

The B matrix is used to show which variables the control is applied. The control matrix B has the general form;

$$B = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ b_{11} & b_{12} \dots & b_{1n} \\ b_{21} & b_{22} \dots & b_{2n} \end{bmatrix} \quad (2.66)$$

where the number of columns n is determined by the size of the control vector \bar{u} . The elements b_{ij} are determined by the

physical implementation of the control. The control matrix B has four rows because the state variable has four elements.

The control vector defines which variables are fed back and how much is fed back, hence \bar{u} is defined as;

$$\bar{u} = K(\tau)\bar{\eta} \quad (2.67)$$

where $K(\tau)$ is a time dependent gain matrix of the form;

$$K(\tau) = \begin{bmatrix} K_{11}(\tau) & K_{12}(\tau) & K_{13}(\tau) & K_{14}(\tau) \\ K_{21}(\tau) & K_{22}(\tau) & K_{23}(\tau) & K_{24}(\tau) \\ \vdots & \vdots & \vdots & \vdots \\ K_{n1}(\tau) & K_{n2}(\tau) & K_{n3}(\tau) & K_{n4}(\tau) \end{bmatrix} \quad (2.68)$$

The number of rows is an option of the designer and equals the order of the control vector \bar{u} .

For now a simple scalar control will be demonstrated, ($n = 1$). The B matrix will assumed to be;

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad (2.69)$$

Then the control term $F^{-1}B\bar{u}$ of equation (2.65) becomes;

$$F^{-1}B\bar{u} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & . & . & \vdots \\ f_{31} & . & . & \vdots \\ f_{41} & \dots & & f_{44} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix} \bar{\eta} \quad (2.70a)$$

or

$$F^{-1}B\bar{u} = \begin{bmatrix} f_{13} + f_{14} \\ f_{23} + f_{24} \\ f_{33} + f_{34} \\ f_{43} + f_{44} \end{bmatrix} \begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix} \bar{\eta} \quad (2.70b)$$

where f_{ij} represents the elements of the F inverse matrix.

Further multiplication and insertion into equation (2.65),

where $R_1 = f_{13} + f_{14}$, gives;

$$\bar{\eta}' = \begin{bmatrix} K_1 R_1 + \lambda_1 & K_2 R_1 & K_3 R_1 & K_4 R_1 \\ K_1 R_2 & K_2 R_2 + \lambda_2 & K_3 R_2 & K_4 R_2 \\ K_1 R_3 & K_2 R_3 & K_3 R_3 + \lambda_3 & K_4 R_3 \\ K_1 R_4 & K_2 R_4 & K_3 R_4 & K_4 R_4 + \lambda_4 \end{bmatrix} \bar{\eta} \quad (2.71)$$

This represents the controlled system using scalar control.

Suppose one mode is unstable, for example λ_1 is positive.

By choosing $K_2 = K_3 = K_4 = 0$, equation (2.71) reduces to;

$$\bar{\eta}' = \begin{bmatrix} K_1(f_{13} + f_{14}) + \lambda_1 & 0 & 0 & 0 \\ K_1(f_{23} + f_{24}) & \lambda_2 & 0 & 0 \\ K_1(f_{33} + f_{34}) & 0 & \lambda_3 & 0 \\ K_1(f_{43} + f_{44}) & 0 & 0 & \lambda_4 \end{bmatrix} \bar{\eta} \quad (2.72)$$

Thus the three stable modes remain the same, and the unstable mode can be changed by proper selection of $K_1(\tau)$.

The differential equation for the first mode is;

$$\eta_1' = \left[K_1(\tau)(f_{13}(\tau) + f_{14}(\tau)) + \lambda_1 \right] \eta_1 \quad (2.73)$$

Using an integrating factor,

$$\mu = \exp \left| -\int_0^T (\lambda_1 + K_1(t)(f_{13}(t) + f_{14}(t)))dt \right| \quad (2.74)$$

the solution of (2.73) is given by;

$$\eta_1 = \eta_{10} \exp \left| \int_0^T (C + R(t))dt \right| \quad (2.75)$$

where here η_{10} is the initial value of η_1 , C is the constant part of the exponential of equation (2.74), and the term $R(t)$ is the time varying part. Now by letting K_1 be a constant, and expressing the components of F inverse in Fourier series form, with a constant and a time periodic part, equation (2.75) reduces to;

$$\eta_1 = \eta_{10} e^C \quad (2.76)$$

with

$$C = \lambda_1 + \alpha K_1 \quad (2.77)$$

where α is the constant part of the Fourier series. Hence by appropriately selected K such that C is less than zero the stability of mode one is guaranteed.

Now suppose there is no constant part of the Fourier series, or that it is too small to be realistically fed back by a constant K . In this case a time varying gain can be used to control the system. The integrating factor of equation (2.74) shows a $K(\tau)$ times a Fourier series. Suppose the first cosine term of the Fourier series is large enough to modify the mode. A constant part can be created by making

the gain;

$$K_1(\tau) = K_{10} \cos \frac{2\pi\tau}{T} \quad (2.78)$$

where K_{10} is constant. When this is multiplied by the Fourier series representation of $(f_{13} + f_{14})$ it gives;

$$K_1(\tau)(f_{13} + f_{14}) = K_{10} \cos \frac{2\pi\tau}{T} \left[\alpha_0 + \alpha_1 \cos \frac{2\pi\tau}{T} + \alpha_2 \cos \frac{4\pi\tau}{T} + \dots + \beta_1 \sin \frac{2\pi\tau}{T} + \beta_2 \sin \frac{4\pi\tau}{T} + \dots \right] \quad (2.79)$$

or

$$K_1(\tau)(f_{13} + f_{14}) = \alpha_0 K_{10} \cos \frac{2\pi\tau}{T} + \alpha_1 K_{10} \cos^2 \left(\frac{2\pi\tau}{T} \right) + \alpha_2 K_{10} \cos \frac{2\pi\tau}{T} \cos \frac{4\pi\tau}{T} + \dots \quad (2.80)$$

rearranging the second term in equation (2.80);

$$\alpha_1 K_{10} \cos^2 \left(\frac{2\pi\tau}{T} \right) = \frac{1}{2} \alpha_1 K_{10} (1 + \cos \frac{4\pi\tau}{T}) \quad (2.81)$$

Therefore the constant part of equation (2.75) becomes;

$$C = \lambda_1 + \frac{1}{2} \alpha_1 K_{10} \quad (2.82)$$

Since the rest of the time periodic terms integrated over the period still equals zero, the stability can again be insured by selecting the appropriate K_{10} to be used with the time periodic gain. Of course any sine or cosine function

can be used depending on the Fourier coefficient the designer wishes to use. The form of equation (2.82) remains the same.

Now suppose there are two unstable modes in equation (2.71). A scalar control may again be used but the problem becomes more involved. Suppose further that the unstable characteristic exponents are complex conjugates. Applying the scalar control equation (2.71) becomes;

$$\bar{\eta}' = \begin{bmatrix} K_1(f_{13} + f_{14}) + \sigma & K_2(f_{13} + f_{14}) + \omega & 0 & 0 \\ K_1(f_{23} + f_{24}) - \omega & K_2(f_{23} + f_{24}) + \sigma & 0 & 0 \\ K_1(f_{33} + f_{34}) & K_2(f_{33} + f_{34}) & \lambda_3 & 0 \\ K_1(f_{43} + f_{44}) & K_2(f_{43} + f_{44}) & 0 & \lambda_4 \end{bmatrix} \bar{\eta} \quad (2.83)$$

where σ is the real part of the complex conjugate and ω is the imaginary part. Again the stability of modes η_3 and η_4 are unaffected. The η_1 and η_2 differential equations are;

$$\begin{aligned} \eta_1' &= (K_1(f_{13} + f_{14}) + \sigma) \eta_1 + (K_2(f_{13} + f_{14}) + \omega) \eta_2 \\ \eta_2' &= (K_1(f_{23} + f_{24}) - \omega) \eta_1 + (K_2(f_{23} + f_{24}) + \sigma) \eta_2 \end{aligned} \quad (2.84)$$

Unfortunately this set of equations does not lend itself to the same method of analysis as the last case. That is, an integrating factor has not been found to solve this set of equations and thus determine what values of K_1 and K_2 will guarantee stability. However equation (2.84) can be written as a periodic linear system;

$$\bar{\eta}' = X(t) \bar{\eta} \quad (2.85)$$

and then a theorem from Floquet theory, which states (3;236),

$$\rho_1 \rho_2 = \exp \left| \int_0^T \text{tr} \left\{ X(s) \right\} ds \right| \quad (2.86)$$

can offer some help. Here ρ_i are the characteristic multipliers and $\text{tr}(X)$ is the trace of the X matrix. Taking the log of both sides gives;

$$\ln(\rho_1 \rho_2) = \int_0^T \text{tr} \left\{ X(s) \right\} ds \quad (2.87)$$

from equation (2.84) the trace of the matrix is;

$$\text{tr} \left\{ X(s) \right\} = K_1(f_{13} + f_{14}) + K_2(f_{23} + f_{24}) + 2\sigma \quad (2.88)$$

or breaking it up into constant and periodic parts,

$$\text{tr} \left\{ X(s) \right\} = (K_1 \alpha_{1i} + K_2 \alpha_{2j} + 2\sigma) + R(\tau) \quad (2.89)$$

where the second subscript on the α 's represents the coefficient of the Fourier series term yielding a constant part. These are determined by the form of the gains K_1 and K_2 . Integrating the periodic part over one period reduces to zero while the constant part leaves;

$$\ln(\rho_1 \rho_2) = (K_1 \alpha_{1i} + K_2 \alpha_{2j} + 2\sigma)T \quad (2.90)$$

or by rearranging,

$$\frac{1}{T} \ln(\rho_1 \rho_2) = \frac{1}{T} \ln \rho_1 + \frac{1}{T} \ln \rho_2 = K_1 \alpha_{1i} + K_2 \alpha_{2j} + 2\sigma \quad (2.91)$$

where $1/T \ln \rho_i$ is recognized as the characteristic exponent

hence,

$$\lambda_1 + \lambda_2 = K_1 \alpha_{11} + K_2 \alpha_{22} + 2\sigma \quad (2.92)$$

Equation (2.91) does not predict the value of the individual characteristic exponents λ_1 and λ_2 , but rather, only their sum. Naturally their sum needs to be negative for stability, but making their sum negative does not guarantee that both values will be negative. It is desirable to be able to predict what values of K_1 and K_2 will guarantee both modes becoming stable, but as of yet this is not possible. The best that can be done, using scalar control, is to use the method described, to help search for gains that will result in a stable system.

There is a method that will predict the value of the controlled characteristic exponents. This method uses a multiple input controller, which enlarges the B and K matrices of equation (2.70). The following is a demonstration of a multiple input control system used to control two unstable modes. First let the B matrix be,

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (2.93)$$

and the K matrix;

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \end{bmatrix} \quad (2.94)$$

Hence,

$$F^{-1}BK\bar{\eta} = \begin{bmatrix} F^{-1} \\ K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \end{bmatrix} \bar{\eta} \quad (2.95)$$

Therefore the controlled system becomes,

$$\bar{\eta}' = \begin{bmatrix} K_{11}f_{13} + K_{21}f_{14} + \sigma & K_{12}f_{13} + K_{22}f_{14} + \omega & 0 & 0 \\ K_{11}f_{23} + K_{21}f_{24} - \omega & K_{12}f_{23} + K_{22}f_{24} + \sigma & 0 & 0 \\ K_{11}f_{33} + K_{21}f_{34} & K_{12}f_{33} + K_{22}f_{34} & \lambda_3 & 0 \\ K_{11}f_{43} + K_{21}f_{44} & K_{12}f_{43} + K_{22}f_{44} & 0 & \lambda_4 \end{bmatrix} \bar{\eta} \quad (2.96)$$

where setting $K_{13} = K_{23} = K_{14} = K_{24} = 0$ gives the above result. Since the stability of the λ_3 and λ_4 modes is not affected by the control, the stability of the controlled λ_1 and λ_2 is determined by the solution to;

$$\begin{bmatrix} \eta_1' \\ \eta_2' \end{bmatrix} = \begin{bmatrix} K_{11}f_{13} + K_{21}f_{14} + \sigma & K_{12}f_{13} + K_{22}f_{14} + \omega \\ K_{11}f_{23} + K_{21}f_{24} - \omega & K_{12}f_{23} + K_{22}f_{24} + \sigma \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \quad (2.97)$$

To determine the values of the K's that will guarantee stability of the system, first set the off diagonal terms to zero. This uncouples the two modes and results in,

$$\begin{aligned} K_{11}f_{23} + K_{21}f_{24} &= \omega \\ K_{12}f_{13} + K_{22}f_{14} &= -\omega \end{aligned} \quad (2.98)$$

solving for K_{11} and K_{12} gives,

$$\begin{aligned} K_{11} &= ((\omega - K_{21}f_{24})/f_{23}) \\ K_{12} &= ((-\omega - K_{22}f_{14})/f_{13}) \end{aligned} \quad (2.99)$$

Next, set the diagonal terms to desired constants 'a' and 'b',

$$\begin{aligned} \sigma + K_{11}f_{13} + K_{21}f_{14} &= a \\ \sigma + K_{12}f_{23} + K_{22}f_{24} &= b \end{aligned} \quad (2.100)$$

solving for K_{21} and K_{22} yields;

$$\begin{aligned} K_{21} &= [(a - \sigma) - K_{11}f_{13}]/f_{14} \\ K_{22} &= [(b - \sigma) - K_{12}f_{23}]/f_{24} \end{aligned} \quad (2.101)$$

Using equation (2.99) in equation (2.101) gives the K's as;

$$\begin{aligned} K_{11}(\tau) &= \frac{\omega f_{14} - (a - \sigma)f_{24}}{f_{14}f_{23} - f_{13}f_{24}} \\ K_{12}(\tau) &= \frac{-\omega f_{24} - (b - \sigma)f_{14}}{f_{13}f_{24} - f_{14}f_{23}} \\ K_{21}(\tau) &= \frac{(a - \sigma)f_{23} - \omega f_{13}}{f_{14}f_{23} - f_{13}f_{24}} \\ K_{22}(\tau) &= \frac{(b - \sigma)f_{13} + \omega f_{23}}{f_{13}f_{24} - f_{14}f_{23}} \end{aligned} \quad (2.102)$$

These K's are time varying and can be expressed in Fourier series form. Thus the K matrix with the above values of K and the B matrix of equation (2.93) gives the final system;

$$\bar{\eta}' = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix} \bar{\eta} \quad (2.103)$$

where a and b are selected to be negative and thus stable.

CHAPTER 3

RESULTS

In this chapter the results of various examples using the theory from the last chapter will be presented and explained. These examples were done using various computer programs to solve for the equations given in the theory chapter. These programs also assisted in selecting the control system to be used, implementing the control, and simulating the results. Presented first is a listing and classification of the stability parameters for various uncontrolled satellite configurations. Next the results of using scalar control to control a system with two unstable modes, is presented. Finally the case of two unstable modes is solved using the multiple input control.

PARAMETER SPACE K vs α

The basic system was expressed in the linear form;

$$\bar{x}' = A(\tau)\bar{x} \quad (3.1)$$

where $A(\tau)$ is a periodic matrix whose elements are given by equations (2.26). These elements are functions of the parameters K , α , and e . Hence the stability is uniquely determined by the given values of these three parameters. In the article by Kane and Barba (4;405) a parameter space is shown of K

verses α for various values of eccentricity. In this study a plot of K verses α was made using an eccentricity of 0.5. The plot originates from a listing of the characteristic exponents for each pair of K and α coordinates, and also a classification of the stability represented by those characteristic exponents. These results are listed in Table 3.1. The system is stable if all real parts of the characteristic exponents are less than zero. There are many cases where the real part is equal to zero, or very close to zero, these cases are considered stable since this represents motion that is not increasing without bound. The zero real part is due to the conservative nature of the system. When the characteristic exponents are complex, the motion will be oscillatory with the frequency of oscillation dependent on the size of the real and imaginary parts. The damping of the system is the negative of the real part divided by the natural frequency of the system. Hence if the real part is negative the damping is positive and the motion dies out in time. If the real part is positive the damping is negative and the motion increases with time. The size of the real part determines how fast the motion either increases or decreases. For example, a large positive real part causes the motion to increase faster than a small positive real part.

TABLE 3.1 - CHARACTERISTIC EXPONENTS FOR K VS α , ($e = 0.5$)

| $\alpha =$ | K = 1.0 | | | K = 0.8 | | |
|------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| | | | | | | |
| 3.0 | 0.0000 +/- j0.1395 | 0.0000 +/- j0.3525 | 0.0000 +/- j0.1489 | 0.0000 +/- j0.4907 | | |
| 2.0 | 0.0000 +/- j0.1174 | 0.0000 +/- j0.4937 | 0.0000 +/- j0.2241 | 0.0000 +/- j0.2820 | | |
| 1.0 | 0.0000 +/- j0.1549 | 0.0000 +/- j0.3766 | 0.0000 +/- j0.1701 | 0.0000 +/- j0.4595 | | |
| 0.0 | 0.1878 | -0.1878 | 0.0000 +/- j0.1444 | 0.0525 | -0.0525 | 0.0000 +/- j0.2362 |
| -1.0 | 0.6718 | -0.6718 | 0.0000 +/- j0.3728 | 0.6072 | -0.6072 | 0.0000 +/- j0.2066 |
| -2.0 | 0.3076 | -0.3076 | 0.0000 +/- j0.2048 | 0.0000 +/- j0.1272 | 0.0000 +/- j0.4209 | |
| -3.0 | 0.0953 | -0.0953 | 0.0000 +/- j0.0327 | 0.0854 | -0.0854 | 0.0000 +/- j0.0129 |

| $\alpha =$ | K = 0.6 | | | K = 0.4 | | |
|------------|--------------------|--------------------|--------------------|--------------------|---------------------|--------------------|
| | | | | | | |
| 3.0 | 0.0000 +/- j0.1519 | 0.0000 +/- j0.3791 | 0.0000 +/- j0.2559 | 0.0000 +/- j0.1447 | | |
| 2.0 | 0.0000 +/- j0.0486 | 0.0000 +/- j0.3082 | 0.0000 +/- j0.1368 | 0.0000 +/- j0.4276 | | |
| 1.0 | 0.0885 | -0.0885 | 0.0000 +/- j0.2189 | 0.0000 +/- j0.2980 | 0.0000 +/- j0.0349 | |
| 0.0 | 0.1476 | 0.5549 | -0.5549 | 0.1214 +/- j0.2959 | -0.1214 +/- j0.2959 | |
| -1.0 | 0.5070 | -0.5070 | 0.0000 +/- j0.0175 | 0.2943 | -0.2943 | 0.0000 +/- j0.2163 |
| -2.0 | 0.0000 +/- j0.1206 | 0.0000 +/- j0.4005 | 0.0000 +/- j0.4390 | 0.0000 +/- j0.2173 | | |
| -3.0 | 0.0000 +/- j0.4149 | 0.0000 +/- j0.4414 | 0.0000 +/- j0.2863 | 0.0000 +/- j0.0651 | | |

$\alpha =$ $K = 0.2$ $K = 0.0$

| | | | | | | | | | | | | |
|------|--------|-----|---------|---------|-----|---------|--------|-----|---------|--------|-----|---------|
| 3.0 | 0.0000 | +/- | j0.1215 | 0.0000 | +/- | j0.1099 | 0.0000 | +/- | j0.0000 | 0.0000 | +/- | j0.0000 |
| 2.0 | 0.0000 | +/- | j0.3073 | 0.0000 | +/- | j0.1282 | 0.0000 | +/- | j0.0000 | 0.0000 | +/- | j0.0000 |
| 1.0 | 0.0742 | | -0.0742 | 0.0000 | +/- | j0.1762 | 0.0000 | +/- | j0.0000 | 0.0000 | +/- | j0.0000 |
| 0.0 | 0.0710 | | -0.0710 | 0.0000 | +/- | j0.2457 | 0.0000 | | 0.0000 | 0.0000 | | 0.0000 |
| -1.0 | 0.1860 | +/- | j0.4857 | -0.1860 | +/- | j0.4857 | 0.0000 | +/- | j0.0000 | 0.0000 | +/- | j0.0000 |
| -2.0 | 0.0000 | +/- | j0.2793 | 0.0000 | +/- | j0.4692 | 0.0000 | +/- | j0.6030 | 0.0000 | +/- | j0.0000 |
| -3.0 | 0.0000 | +/- | j0.1724 | 0.0000 | +/- | j0.4328 | 0.0000 | +/- | j0.0000 | 0.0000 | +/- | j0.0000 |

 $K = -0.2$ $K = -0.4$

| | | | | | | | | | | | | |
|------|--------|-----|---------|---------|-----|---------|--------|-----|---------|---------|-----|---------|
| 3.0 | 0.0384 | +/- | j0.0932 | -0.0384 | +/- | j0.0932 | 0.4215 | | -0.4215 | 0.0000 | +/- | j0.3602 |
| 2.0 | 0.0983 | +/- | j0.1756 | -0.0983 | +/- | j0.1756 | 0.4385 | | 0.3361 | -0.4385 | | -0.3361 |
| 1.0 | 0.1829 | +/- | j0.3833 | -0.1829 | +/- | j0.3833 | 0.6614 | | 0.3876 | -0.3876 | | -0.6614 |
| 0.0 | 0.5257 | | 0.2162 | -0.2162 | | -0.5257 | 0.7834 | | 0.6559 | -0.6559 | | -0.7834 |
| -1.0 | 0.6017 | +/- | j0.0094 | -0.6017 | +/- | j0.0094 | 0.8141 | +/- | j0.0339 | -0.8141 | +/- | j0.0339 |
| -2.0 | 0.4319 | +/- | j0.3953 | -0.4319 | +/- | j0.3953 | 0.7568 | +/- | j0.2810 | -0.7568 | +/- | j0.2810 |
| -3.0 | 0.2297 | +/- | j0.2971 | -0.2297 | +/- | j0.2971 | 0.4735 | +/- | j0.4128 | -0.4735 | +/- | j0.4128 |

| $\alpha =$ | K = -0.6 | | | | K = -0.8 | | | |
|------------|------------|---------|-------------|---------|------------|---------|-------------|---------|
| | | | | | | | | |
| 3.0 | 0.5384 | 0.3735 | -0.5384 | -0.3735 | 1.2424 | 0.6066 | -0.6066 | -1.2424 |
| 2.0 | 0.8482 | 0.5259 | -0.5259 | -0.8482 | 1.2570 | 0.7543 | -0.7542 | -1.2430 |
| 1.0 | 1.0741 | 0.5117 | -0.5117 | -1.0744 | 1.1584 | 0.9504 | -0.9506 | -1.1524 |
| 0.0 | 0.9275 +/- | j0.3540 | -0.9275 +/- | j0.3540 | 1.0840 +/- | j0.2540 | -1.0840 +/- | j0.2540 |
| -1.0 | 0.9702 +/- | j0.0681 | -0.9702 +/- | j0.0681 | 1.0985 +/- | j0.1107 | -1.0985 +/- | j0.1107 |
| -2.0 | 0.9559 +/- | j0.1550 | -0.9559 +/- | j0.1550 | 1.1002 +/- | j0.0117 | -1.1002 +/- | j0.0117 |
| -3.0 | 0.8834 +/- | j0.3668 | -0.8834 +/- | j0.3668 | 1.0896 +/- | j0.1272 | -1.0896 +/- | j0.1272 |

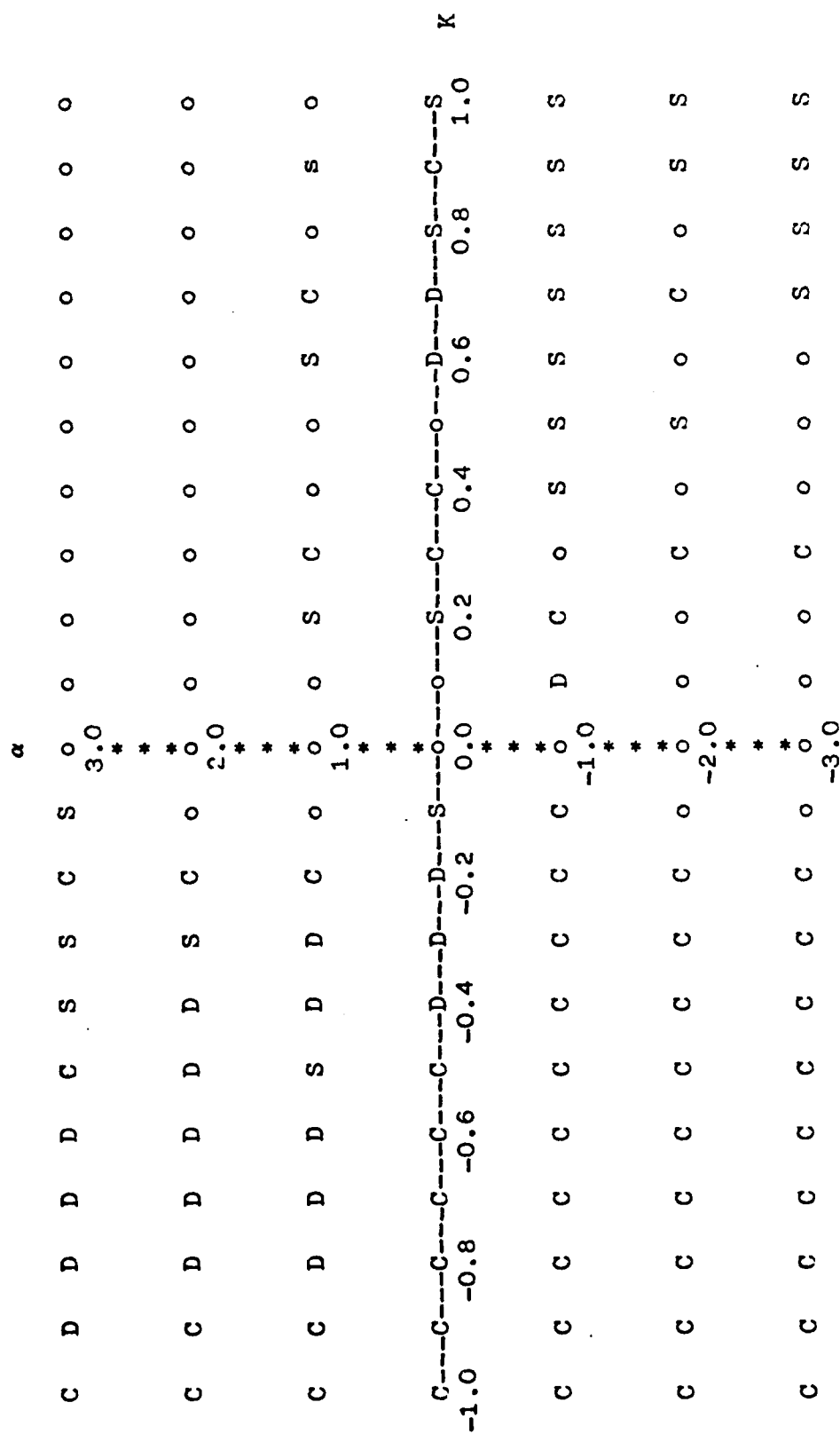
| K = -1.0 | | | | |
|----------|------------|---------|-------------|---------|
| 3.0 | 1.2095 +/- | j0.1618 | -1.2095 +/- | j0.1618 |
| 2.0 | 1.2095 +/- | j0.1618 | -1.2095 +/- | j0.1618 |
| 1.0 | 1.2095 +/- | j0.1618 | -1.2095 +/- | j0.1618 |
| 0.0 | 1.2095 +/- | j0.1618 | -1.2095 +/- | j0.1618 |
| -1.0 | 1.2095 +/- | j0.1618 | -1.2095 +/- | j0.1618 |
| -2.0 | 1.2095 +/- | j0.1618 | -1.2095 +/- | j0.1618 |
| -3.0 | 1.2095 +/- | j0.1618 | -1.2095 +/- | j0.1618 |

Some patterns from Table 3.1 can be recognized as to the stability dependence on the parameters K and α . Specifically the real part of the characteristic exponents approaches zero as the K approaches zero, from either the positive or negative direction. This means the instabilities become less severe. When K equals zero the satellite has the same moments of inertia in all three axes, (i.e. $J=1$ in equation (2.15)). When K equals -1.0 the stability is independent of the spin rate. This case corresponds to J equalling zero, which is not realistically possible. When J is nearly zero the mass of the satellite is distributed very close to the satellite spin axis. For other values of K , the larger the spin parameter, either positive or negative, the instabilities become smaller.

From Table 3.1 many of the characteristic exponents are in similar form. The only cases that are stable are those where the real parts all equal zero. The most prevalent unstable case is the one involving two pair of complex conjugates; one set with a negative real part (stable), the other set with positive real part (unstable). Another type of instability is the set with a stable set of complex conjugates and two purely real characteristic exponents; one stable and one unstable. An example of this case is when $K = 0.8$, and $\alpha = -1.0$. The third type of instability is when all the characteristic exponents are real with two of them less than zero, and two of them greater than zero. A few of these cases have been difficult to work with due to the

numerical problems in obtaining a periodic solution matrix. Therefore the characteristic exponents in some of these third cases may be erroneous.

These different cases are summarized on Figure 3.1, where the 'o' represents a stable case, 'S' means single unstable mode, 'C' means complex conjugates unstable case, and 'D' represents the dual real unstable case.



o=purely imaginary ; C=complex conjugates ; S=single real ; D=dual real

FIGURE 3.1 - PARAMETER SPACE K VS α

CASE STUDY UNCONTROLLED

Yeakel presented the case of one unstable mode along with a technique to control such a case. A scaler control is adequate for changing this one unstable characteristic exponent to a negative value with predictable results. From the parameter space of Figure 3.1 it is obvious that the majority of unstable cases involve two unstable modes. As explained in the theory chapter a scaler control can not predict where these two unstable characteristic exponents will move.

The stability cannot be insured using a scaler control. The only method to date has been to use the relationship between the sum of the characteristic exponents and the integral of the trace of $A(\tau)$ in searching for an acceptable set of gains to be fed back. The case study being looked at has parameters $K = 0.7$ and $\alpha = 1.0$, ($e = 0.5$). This case represents the most common type of instability, but the methods used to control this case along with the results, are applicable to other similar cases, and to cases where the unstable characteristic exponents are purely real. The characteristic exponents for this case are, (uncontrolled);

$$\lambda_{1,2} = 0.0177 \pm j0.3667 \quad \text{unstable}$$

$$\lambda_{3,4} = -0.0177 \pm j0.3667 \quad \text{stable}$$

The motion of the uncontrolled system is shown in Figures

(3.2) thru (3.10). Figures (3.2) thru (3.5) show the motion of the state variables, defined in equation (2.20). Figure (3.6) shows the motion of the angle ϕ , which is the angle between the satellite spin axis and the orbit normal, and is defined by;

$$\phi = \arccos(\cos\theta_1 \cos\theta_2) \quad (3.2)$$

Figures (3.7) thru (3.10) shows the motion of the modal variables defined by equation (2.57). The modal variable motion can be predicted directly from the characteristic exponents. The state variable motion is a combination of all the modes.

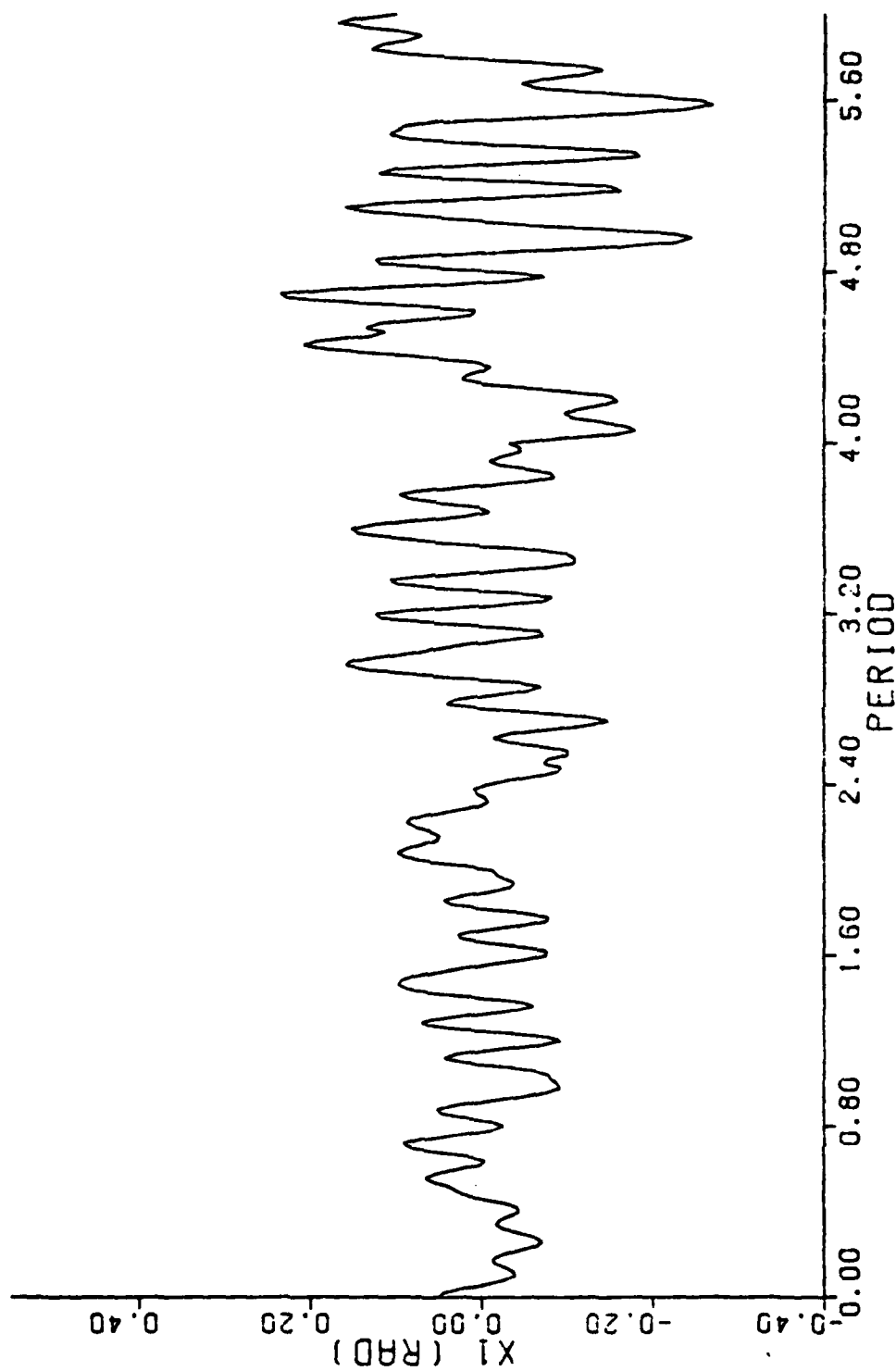


FIGURE 3.2 - Uncontrolled x_1 Response

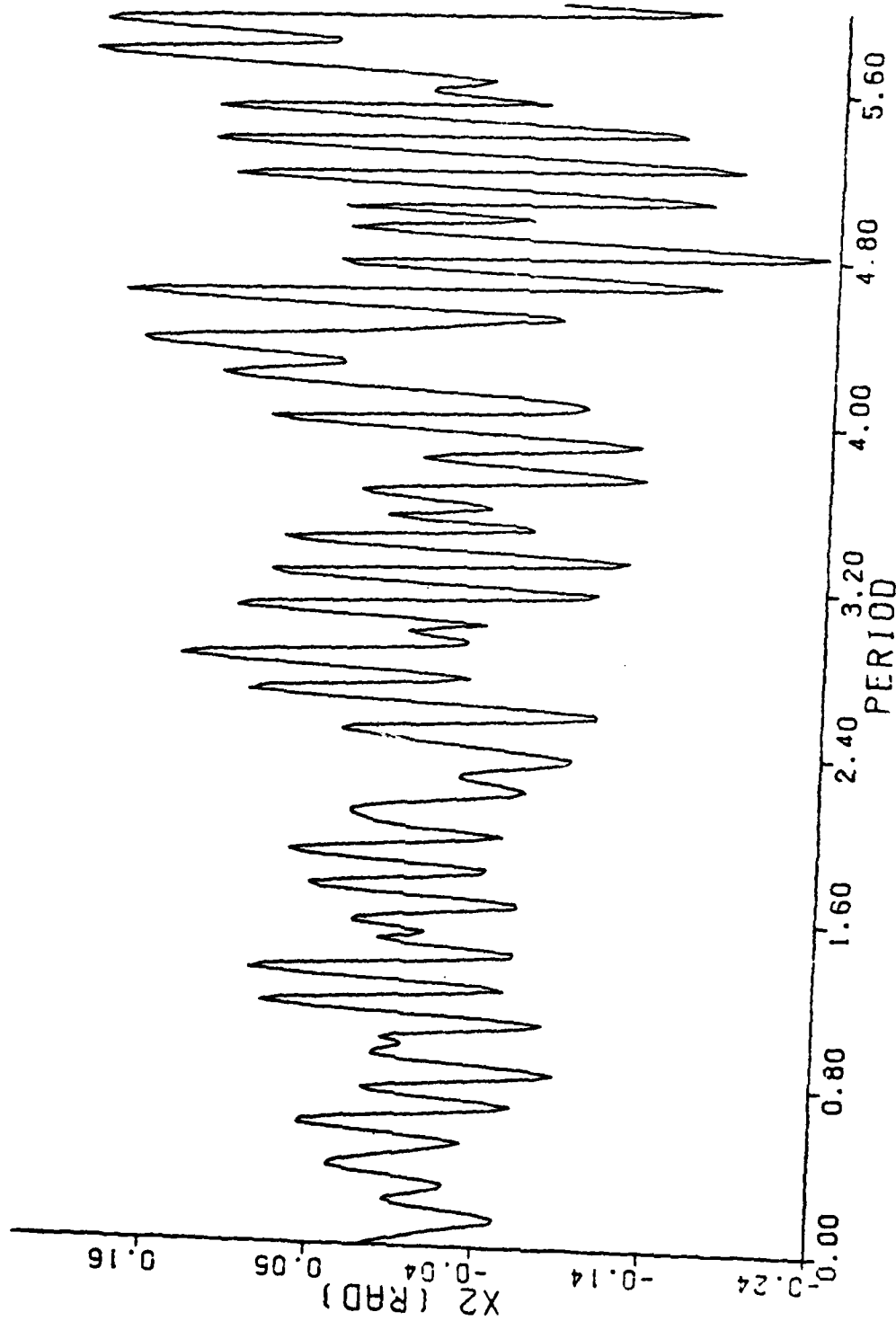


FIGURE 3.3 - Uncontrolled x_2 Response

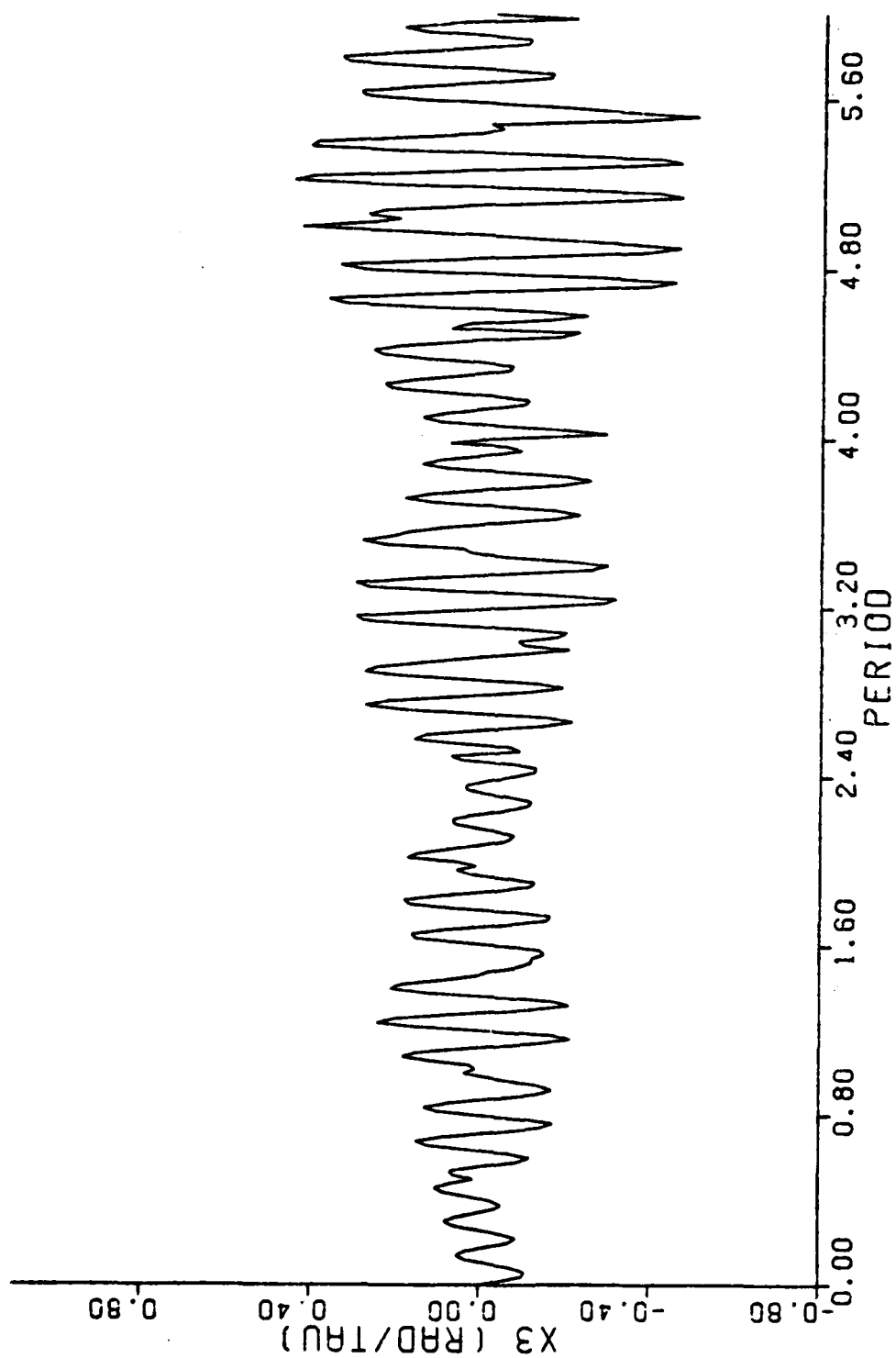


FIGURE 3.4 - Uncontrolled x_3 Response

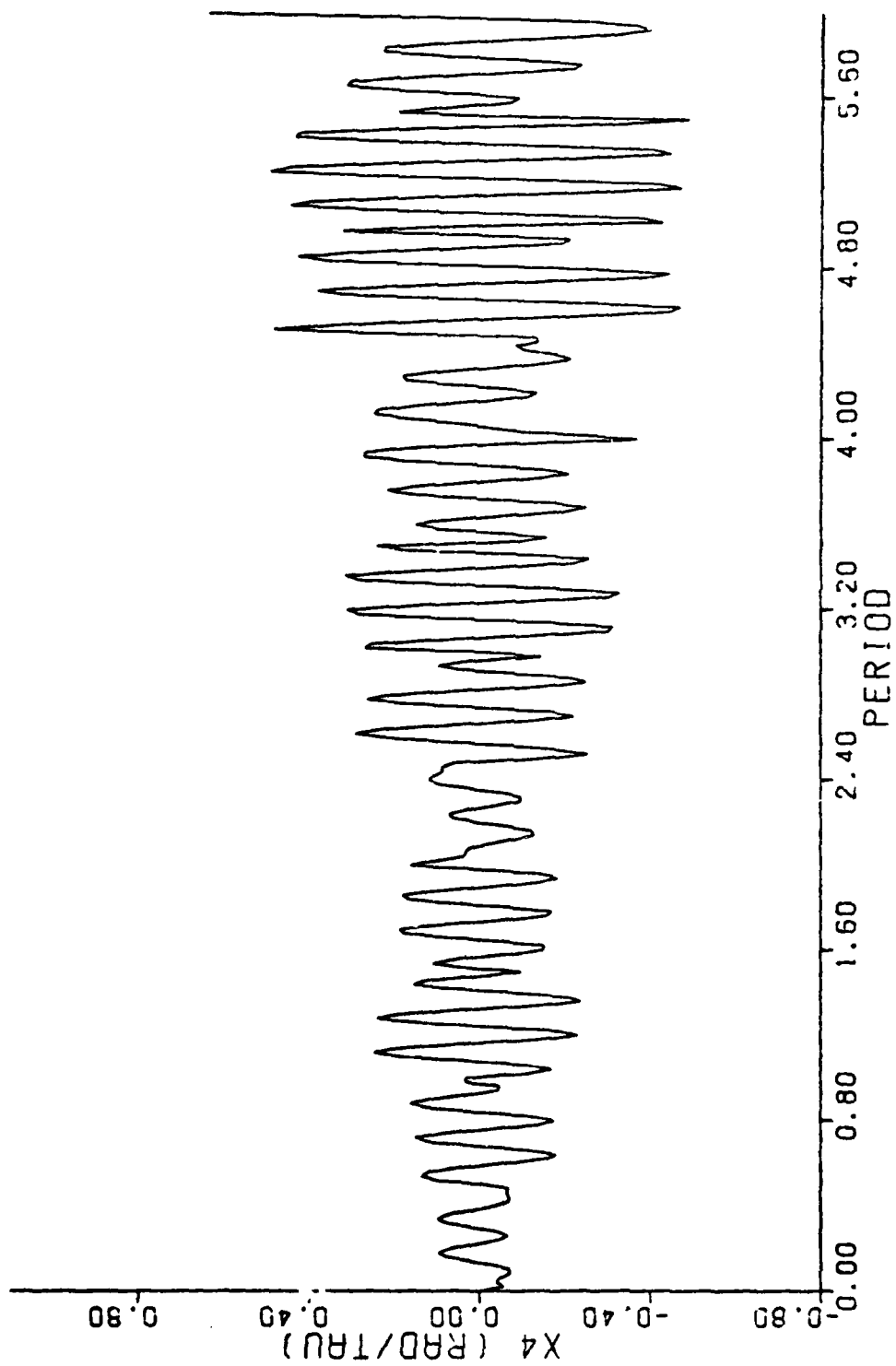


FIGURE 3.5 - Uncontrolled x_4 Response

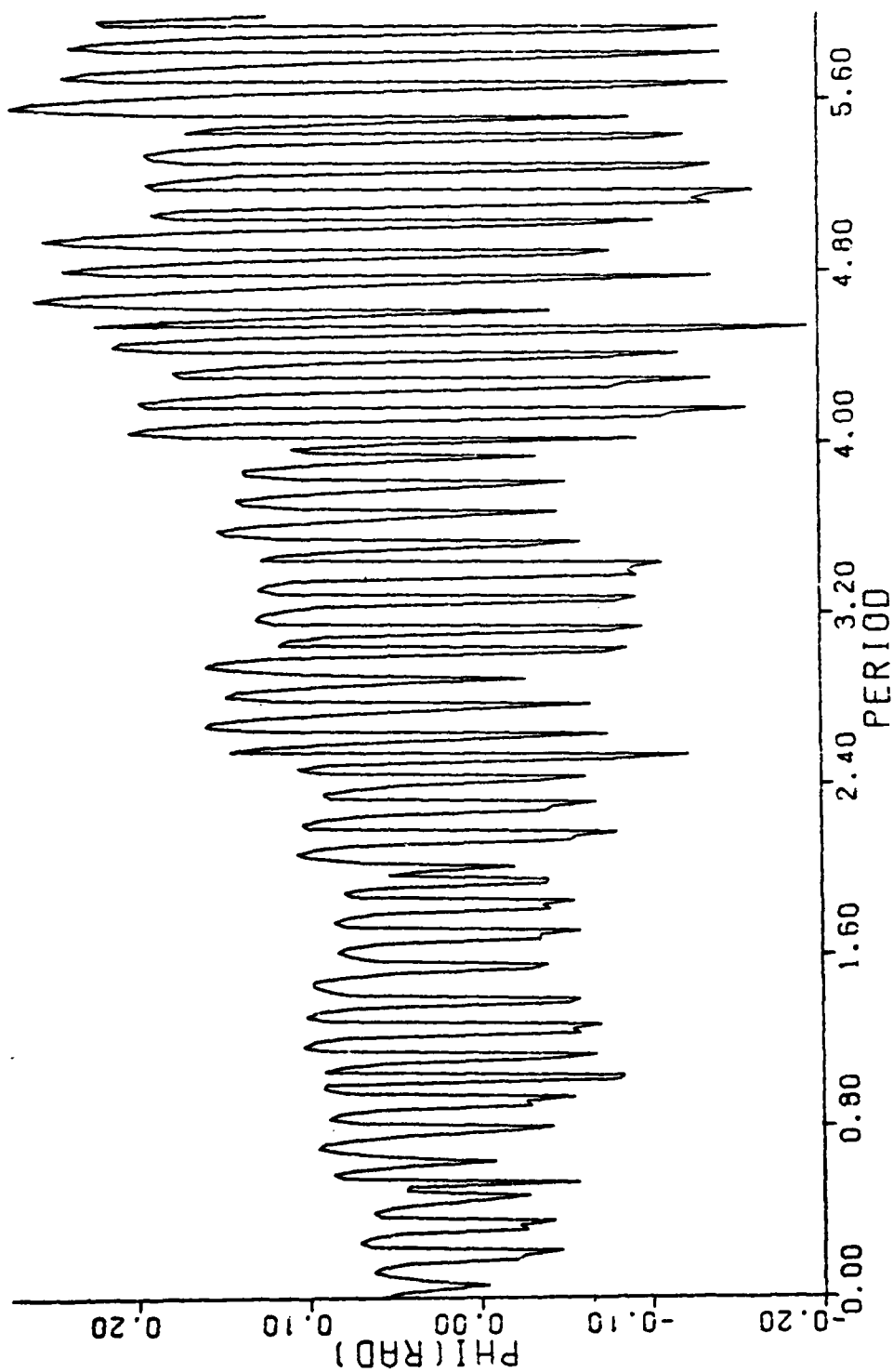


FIGURE 3.6 - Uncontrolled ϕ Response

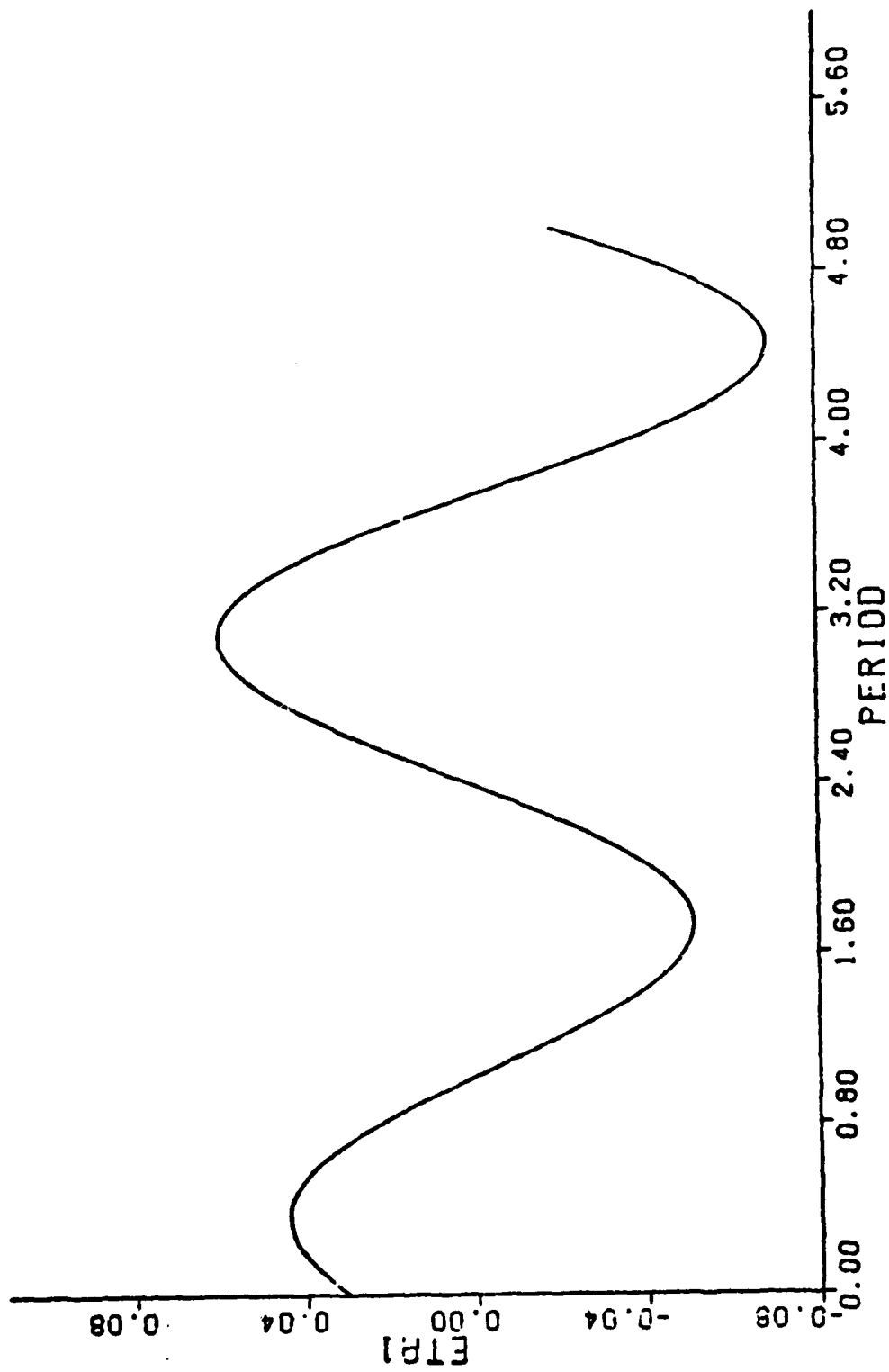


FIGURE 3.7 - Uncontrolled First Mode Response

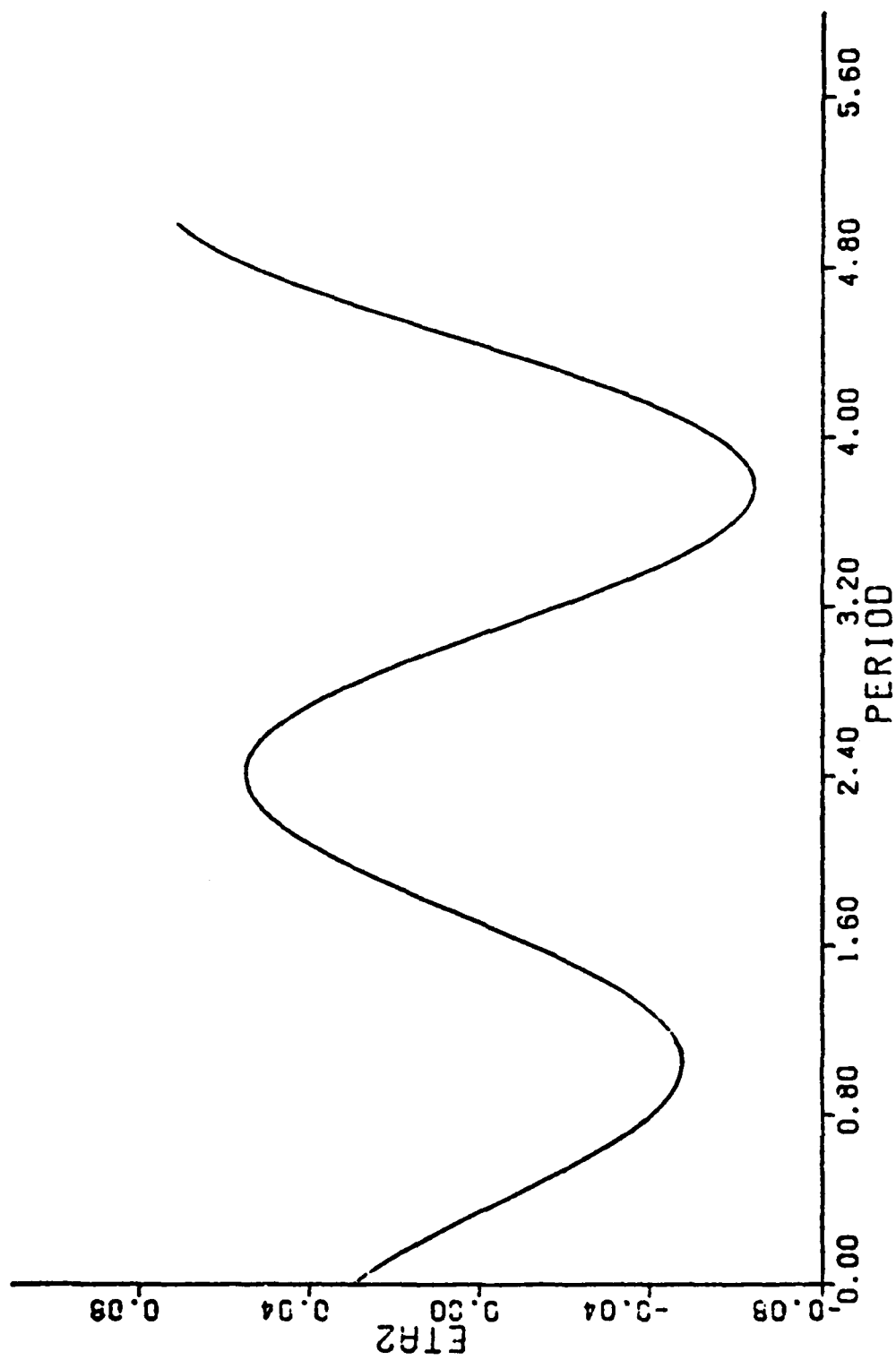


FIGURE 3.8 - Uncontrolled Second Mode Response

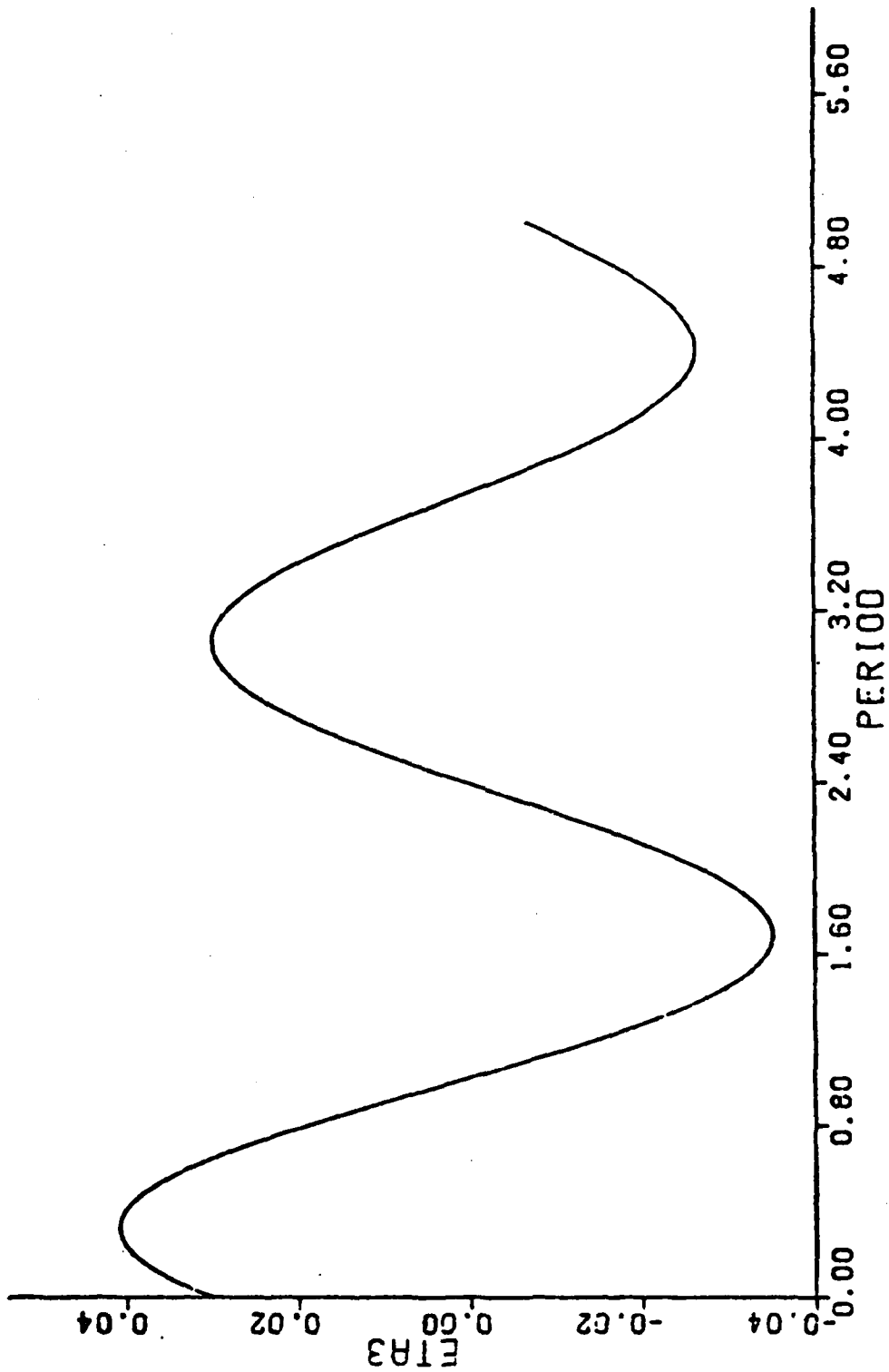


FIGURE 3.9 - Uncontrolled Third Mode Response

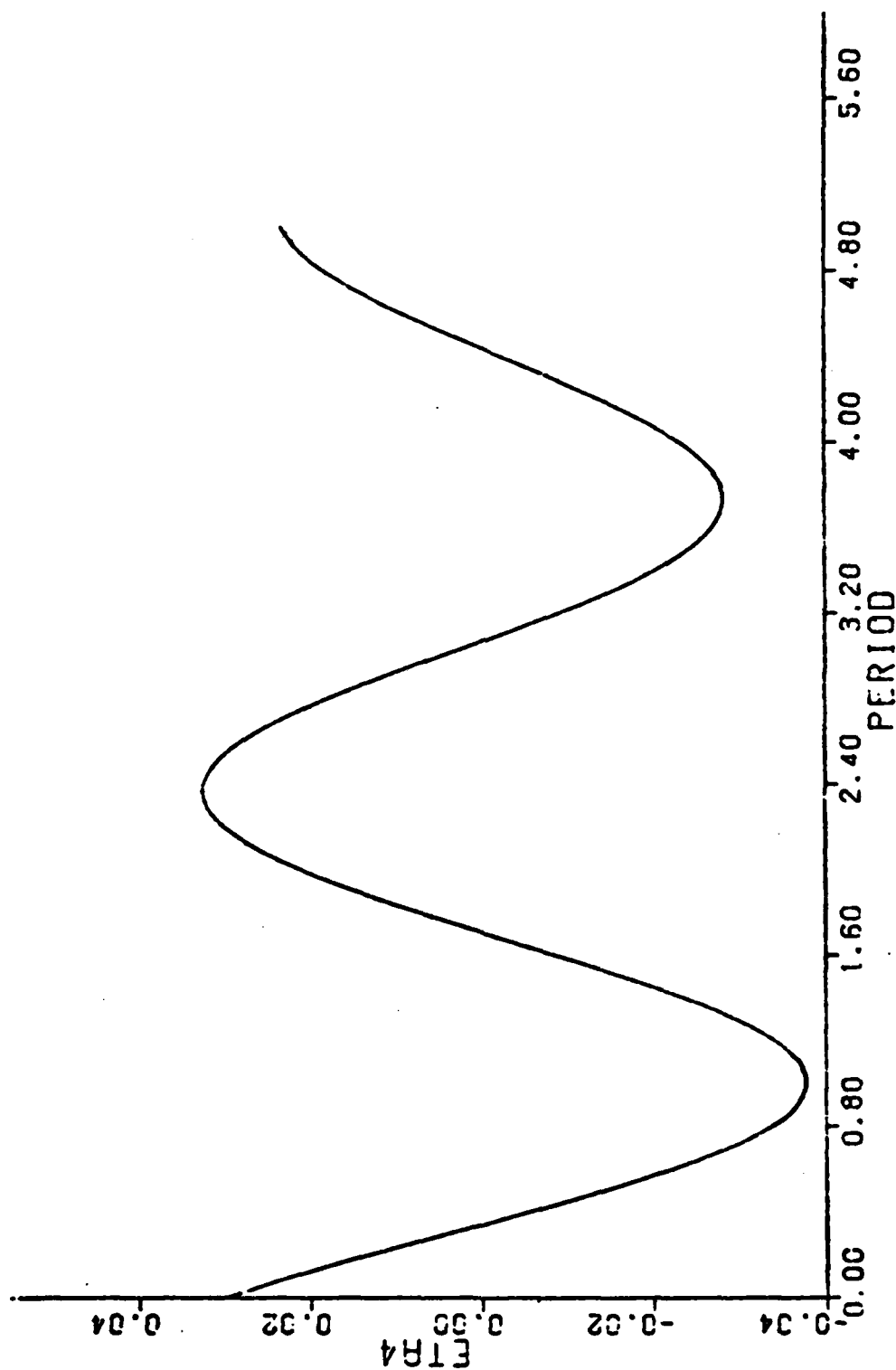


FIGURE 3.10 - Uncontrolled Fourth Mode Response

CASE STUDY SCALER CONTROL

From Floquet theory of chapter 2, a solution to the linear system of equation (3.1) is given by;

$$\psi(\tau) = F(\tau)e^{J\tau} \quad (3.3)$$

where J is given by,

$$J = \begin{bmatrix} 0.0177 & 0.3667 & 0.0000 & 0.0000 \\ -0.3667 & 0.0177 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & -0.0177 & 0.3667 \\ 0.0000 & 0.0000 & -0.3667 & -0.0177 \end{bmatrix} \quad (3.4)$$

The periodic F matrix was found by computer programs which expressed it in Fourier series form. Each element of this 4x4 matrix has 31 cosine terms and 29 sine terms representing its function. A typical element is,

$$\begin{aligned} f_{11}(\tau) = & 0.00 - 0.3239 \cos(2\pi\tau/T) + 0.00 + 0.0688 \cos(6\pi\tau/T) \\ & + \dots + 0.0004 \cos(60\pi\tau/T) - 0.1259 \sin(2\pi\tau/T) \\ & + 0.00 - 0.0326 \sin(6\pi\tau/T) + \dots + \\ & 0.00001 \sin(58\pi\tau/T) \end{aligned}$$

where T is the period of the orbit.

Using scaler control puts equation (2.65) into the form of equation (2.83). Controlling the unstable modes reduces equation (2.83) to the 2x2 system of equation (2.84). The

trace of the matrix of this 2x2 system, used in equation (2.86), is given by;

$$\lambda_1 + \lambda_2 = 2\sigma + K_1(f_{13} + f_{14}) + K_2(f_{23} + f_{24}) \quad (3.5)$$

where the f values in equation (3.5) are the Fourier coefficients of the terms which yield constant parts. For this case the zeroth cosine terms (the constant parts) of the Fourier series are too small to be adequately amplified for control. However the first cosine coefficients are;

$$\begin{aligned} f_{13} &= 0.0 - 1.0027 \cos(2\pi\tau/T) \\ f_{14} &= 0.0 + 0.2689 \cos(2\pi\tau/T) \\ f_{23} &= 0.0 + 0.2919 \cos(2\pi\tau/T) \\ f_{24} &= 0.0 + 0.1128 \cos(2\pi\tau/T) \end{aligned} \quad (3.6)$$

Making the gains time periodic functions such that,

$$\begin{aligned} K_1(\tau) &= G_1 \cos(2\pi\tau/T) \\ K_2(\tau) &= G_2 \cos(2\pi\tau/T) \end{aligned} \quad (3.7)$$

where the G 's are the constant gain values yet to be determined. Now using equation (3.7) and (3.6) in equation (3.5) the constant terms become;

$$\Omega = \lambda_1 + \lambda_2 = 2(0.0177) + G_1(-0.3669) + G_2(0.2024) \quad (3.8)$$

Rearranging the terms in equation (3.8) to express the gains in a ratio gives;

$$\Omega = 0.0354 + G_1(-0.3669 + 0.2024(G_2/G_1)) \quad (3.9)$$

The form of equation (3.9) allows a specific gain ratio to be selected and then a value of G_1 to be varied to achieve desired values of Ω , the sum of the controlled characteristic exponents. As an example, for the ratio of $R = -3.0$, if $G_1 = 1.0$ then $G_2 = -3.0$. From equation (3.9) Ω becomes,

$$\Omega = 0.0354 + (1.0)(-0.3669 + 0.2024(-3.0)) = -0.9387$$

Using Floquet theory the new characteristic exponents of the controlled system are,

$$\lambda_1 = -0.6883 \quad \lambda_2 = -0.2485$$

where the sum of these characteristic exponents is -0.9368 . These values show the system has been stabilized, and since they are purely real there is no oscillation in these modes. There is still oscillation in the other set of stable characteristic exponents, hence the angle ϕ oscillates as shown in Figure (3.11). For the same ratio $R = -3.0$, if $G_1 = 0.6$ then $G_2 = -1.8$ and the predicted sum would be -0.5491 and the new characteristic exponents are,

$$\lambda_{1,2} = -0.2742 \pm j0.1432$$

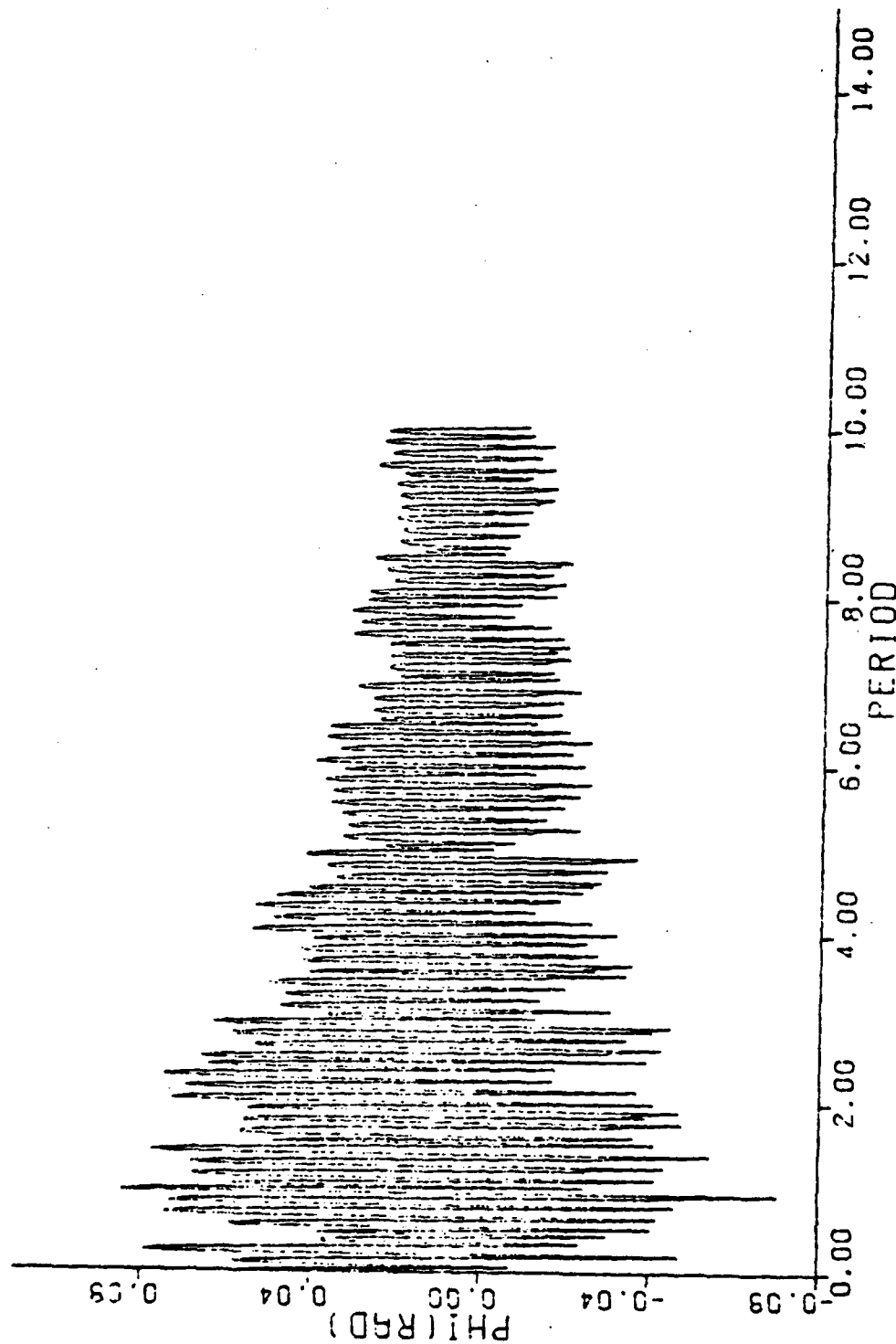


FIGURE 3.11 - Controlled ϕ Response for Scaler Control Case 1

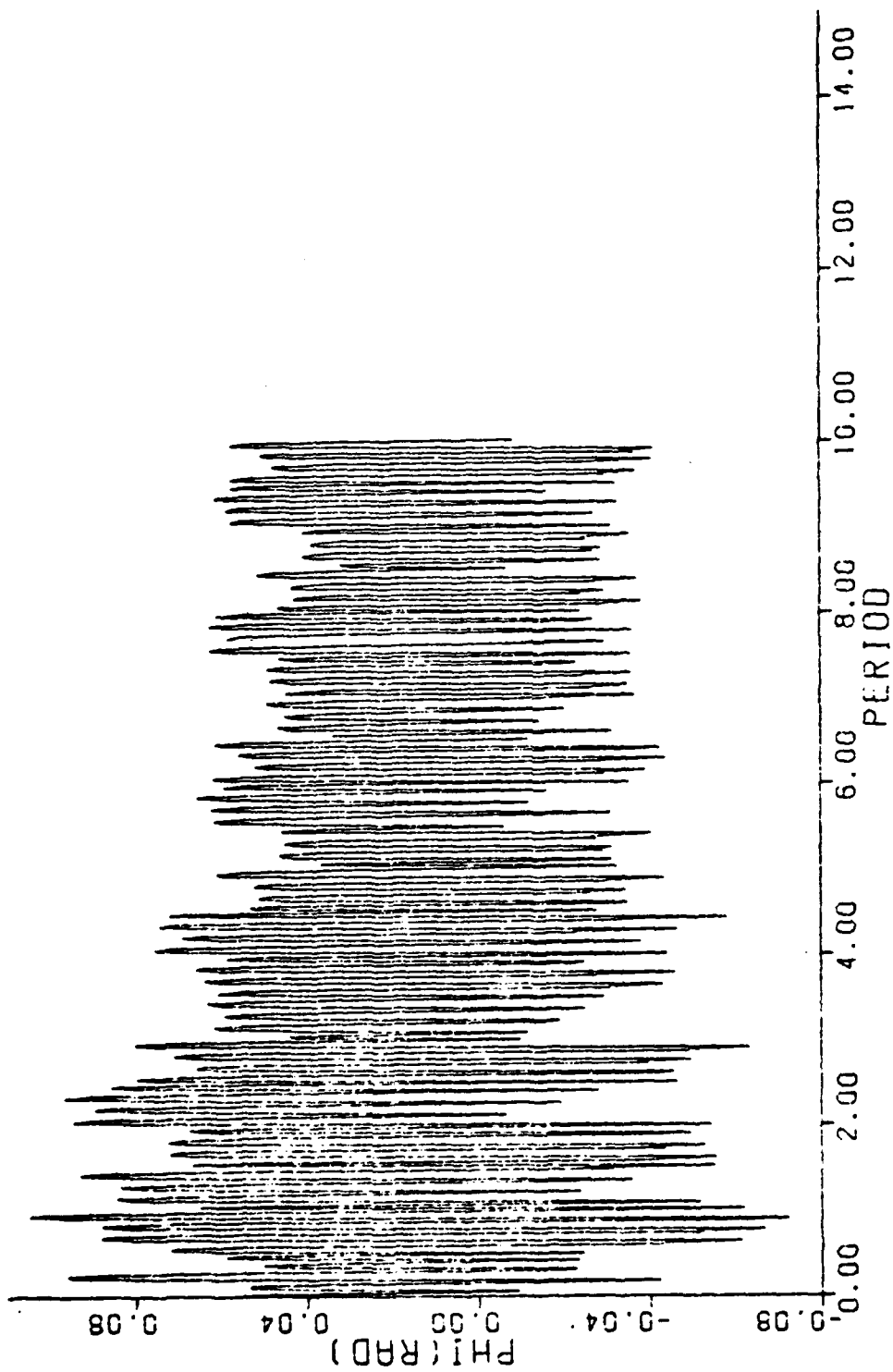


FIGURE 3.12 - Controlled ϕ Response for Scaler Control Case 2

where the sum is -0.5484 . The plot of ϕ for this combination of gains is shown in Figure (3.12). Here there are more oscillations due to the imaginary part of the new characteristic exponents. These two cases both exhibit stable behavior. However, in general for cases such as these, stability is not assured and each case must be evaluated.

By varying the G_1 for specific ratios, a locus of characteristic exponents can be plotted. For different ratios these plots represent a family of curves. These are shown for the case being studied in Figures (3.13) thru (3.18). Note that all curves originate at the unstable uncontrolled characteristic exponents. As the gain is increased from zero, the locus cross the imaginary axis and become stable. This crossover occurs when the trace of the 2×2 system is equal to zero. The locus then breaks in at some point on the real axis and splits, one branch heads toward negative infinity, and the other branch heads toward the origin. This break in point is very important because it can be thought of as the most stable placement for the controlled characteristic exponents, as will be seen. From Figure (3.13) the break in point for a ratio of 0.5 occurs around -0.02 , and from Figure (3.16), the ratio of -10.0 , the break in point is around -0.35 . If the characteristic exponents lie anywhere in the left half plane, they represent a stable configuration. Obviously the closer they are to the real axis, the less the oscillation, and the further they are from the imaginary axis the quicker the motion dies out. The break in point is the point that is furthest from the

FIGURE 3.13 - Locus of Characteristic
Exponents for $R = 0.5$

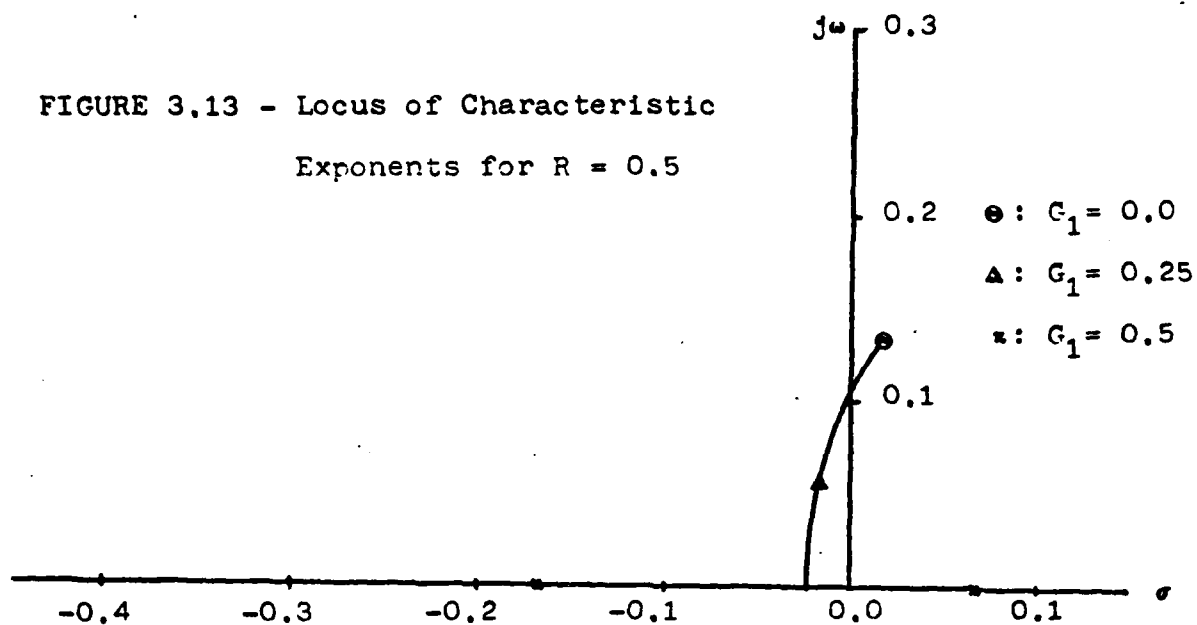


FIGURE 3.14 - Locus of Characteristic
Exponents for $R = -0.5$

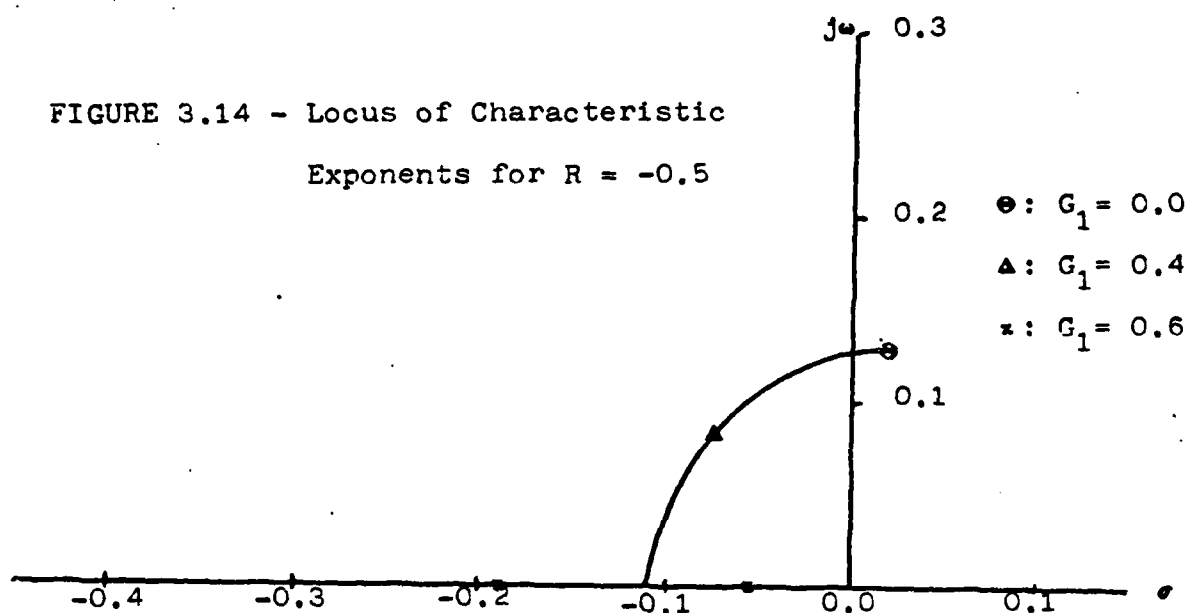


FIGURE 3.15 - Locus of Characteristic
Exponents for $R = -3.0$

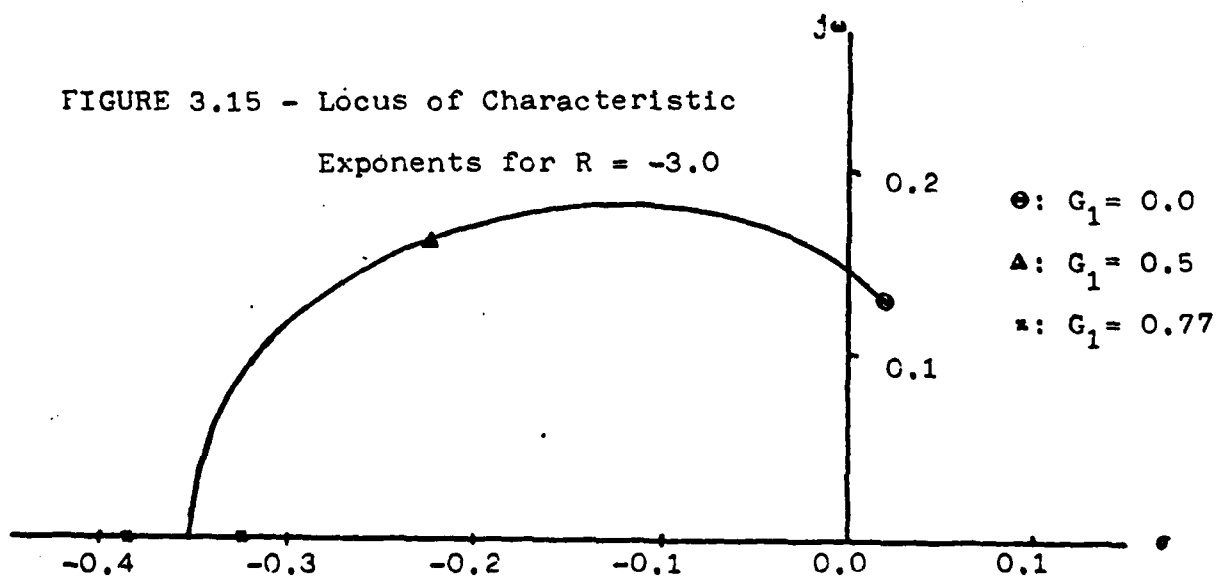


FIGURE 3.16 - Locus of Characteristic
Exponents for $R = -10.0$

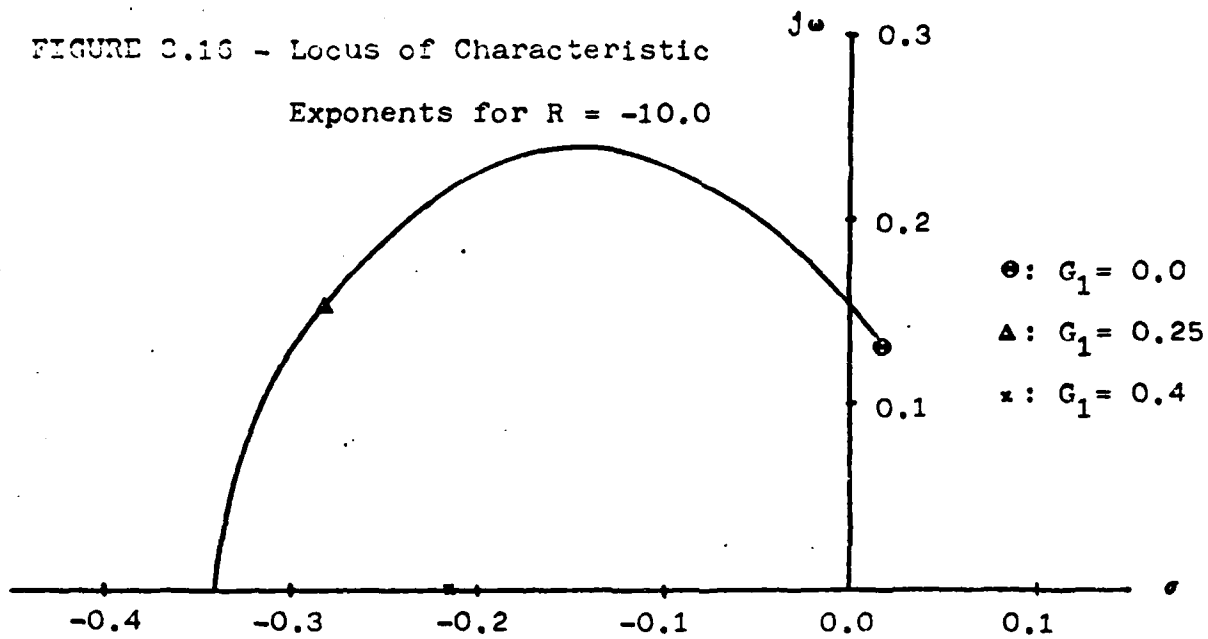


FIGURE 3.17 - Locus of Characteristic
Exponents for $R = -20.0$

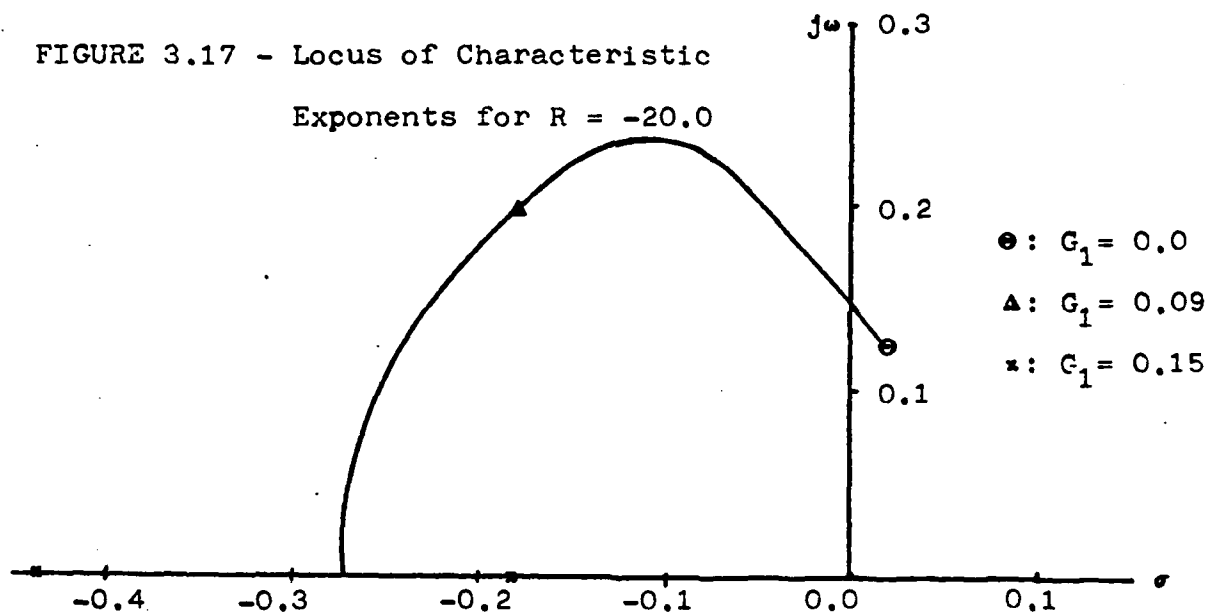
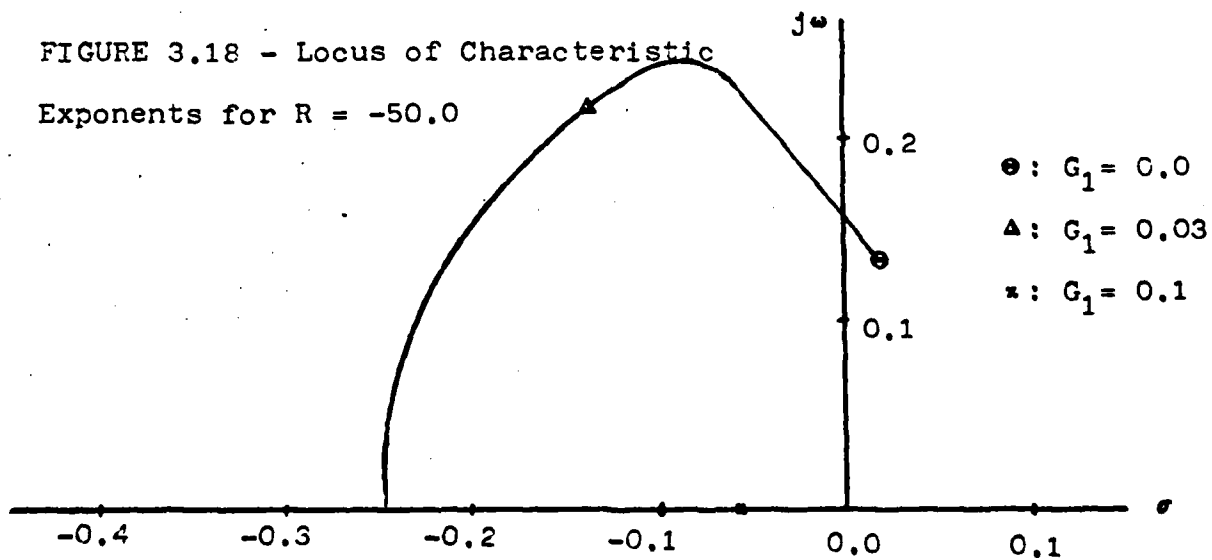


FIGURE 3.18 - Locus of Characteristic
Exponents for $R = -50.0$



imaginary axis and closest to the real axis for both sets of controlled characteristic exponents.

CASE STUDY MULTIPLE INPUT CONTROL

The scalar control used to stabilize a system with two unstable modes is desirable due to the simplicity of the feedback gains used. However since there is no way to accurately predict the placement of the controlled characteristic exponents, another method is needed. The multiple input control system provides more predictable results. Again controlling the unstable modes results in the system described by equation (2.97). The time varying gains were found using equations (2.102). These gains are expressed as Fourier series. The new characteristic exponents were chosen to be;

$$\lambda_1 = -0.75 \quad \lambda_2 = -0.50 \quad (3.10)$$

A computer program found the Fourier series for the gains K_{11} , K_{12} , K_{21} , and K_{22} that would result in the characteristic exponents of equation (3.10). An example of the time varying gains is;

$$\begin{aligned} K_{11}(\tau) = & -1.243 \cos(2\pi\tau/T) + 0.625 \cos(6\pi\tau/T) + \dots \\ & -2.742 \sin(2\pi\tau/T) - 0.111 \sin(6\pi\tau/T) \end{aligned} \quad (3.11)$$

where T is the period of the orbit. A summary of the gain

Fourier coefficients for this case is listed in Table (3.2). The gains given in this case yield the new characteristic exponents of;

$$\lambda_1 = -0.7685 \quad \lambda_2 = -0.4777$$

where the error from the predicted values, is due to the numerical inaccuracies of the computer. More accurate results would have been achieved if the Fourier series had been taken to more coefficients. The ϕ plot for this set of gains is given in Figure (3.19).

Now if the new characteristic exponents were desired to be at;

$$\lambda_1 = -2.00 \quad \lambda_2 = -1.50$$

the Fourier series of the gains would be as listed in Table (3.3), and the resulting characteristic exponents would be at;

$$\lambda_1 = -2.235 \quad \lambda_2 = -1.319$$

The ϕ plot for these gains is shown in Figure (3.20).

TABLE 3.2 - GAIN FUNCTION FOR MULTIPLE CONTROL CASE 1

$$K_{ij}(\tau) = \sum_{n=1}^{10} A_n \cos(2(n-1)\pi\tau/T) + \sum_{m=1}^{10} B_m \sin(2m\pi\tau/T)$$

| | K_{11} | K_{12} | K_{21} | K_{22} |
|---------------|-------------|-------------|-------------|-------------|
| cosine terms; | | | | |
| A_1 | 0.8419 E-7 | 0.6213 E-7 | 0.1670 E-7 | 0.3438 E-8 |
| A_2 | -1.2429 | -0.6862 | -0.5508 E-1 | -1.1693 |
| A_3 | -0.5849 E-7 | -0.1414 E-7 | 0.6755 E-7 | 0.1566 E-7 |
| A_4 | 0.6248 | 1.3368 | 1.4945 | -0.2849 |
| A_5 | -0.5868 E-7 | -0.4116 E-7 | -0.8632 E-7 | -0.9058 E-7 |
| A_6 | 1.4044 | 1.6458 | 1.6669 | -0.2662 |
| A_7 | -0.1008 E-6 | -0.5761 E-7 | 0.3726 E-7 | 0.5176 E-8 |
| A_8 | -1.2509 | -0.3723 | 0.5329 | -0.4886 |
| A_9 | 0.1498 E-6 | 0.1258 E-6 | 0.2521 E-7 | 0.2693 E-8 |
| A_{10} | -2.1042 | -0.6031 | -0.3944 | -0.6961 |
| sine terms; | | | | |
| B_1 | -2.7417 | -1.6274 | -0.3833 E-2 | 0.2135 |
| B_2 | 0.1118 E-6 | 0.8622 E-7 | 0.6379 E-7 | 0.4292 E-7 |
| B_3 | -0.1112 | -0.1777 | -1.5967 | -1.2184 |
| B_4 | 0.2076 E-7 | 0.1988 E-7 | 0.1357 E-7 | 0.1058 E-7 |
| B_5 | -0.3721 | 0.2358 | 2.0491 | 0.9833 |
| B_6 | -0.1006 E-6 | -0.9000 E-7 | -0.5690 E-7 | -0.4547 E-7 |
| B_7 | 3.2091 | 1.5700 | 0.2321 E-1 | -0.1490 |
| B_8 | -0.5102 E-7 | -0.2927 E-7 | -0.2555 E-7 | -0.2070 E-7 |
| B_9 | -2.3053 | -0.8608 | 0.7180 | -0.3420 E-1 |
| B_{10} | 0.9814 E-7 | 0.7360 E-7 | 0.5153 E-7 | 0.4196 E-7 |

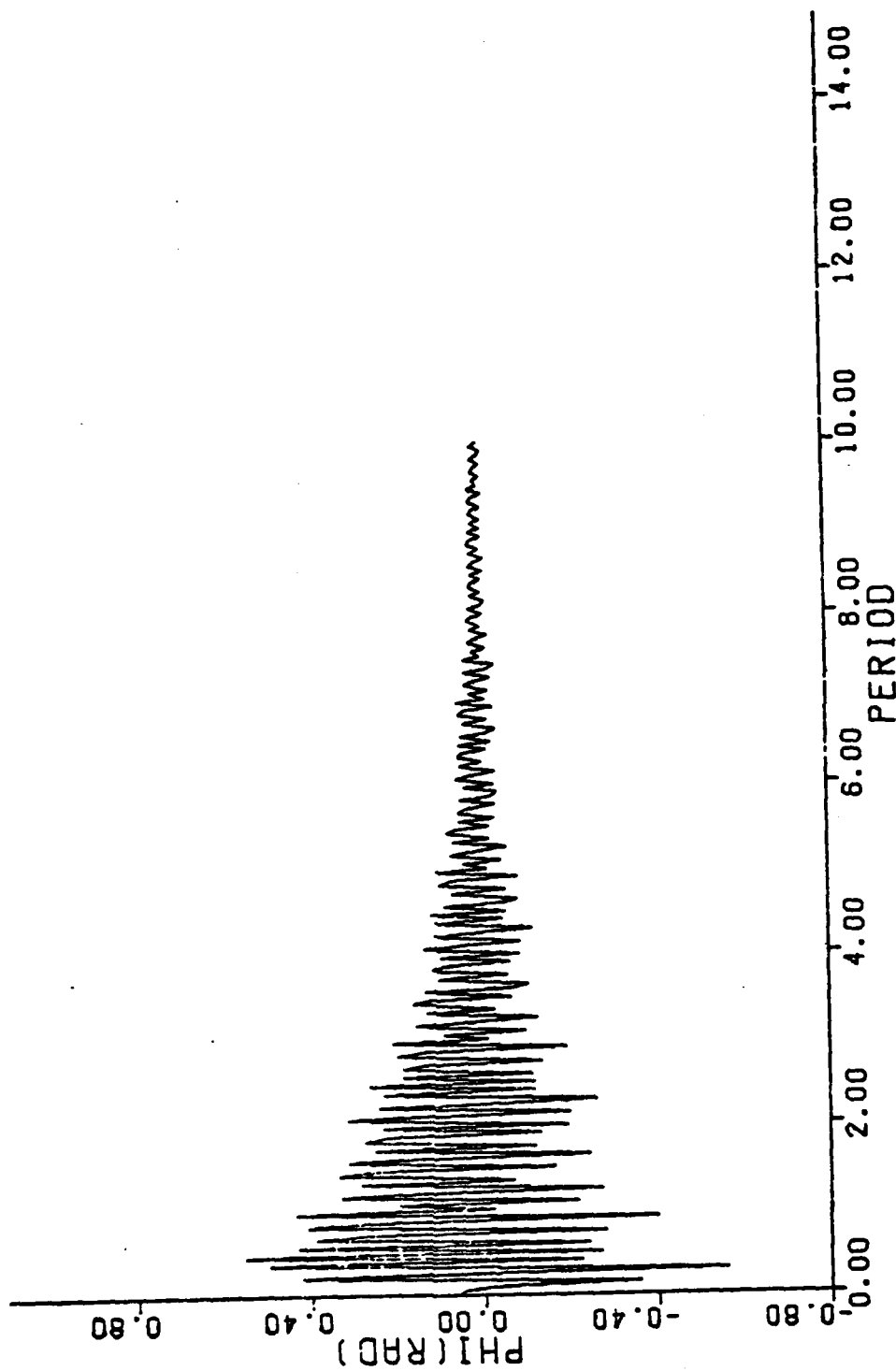


FIGURE 3.19 - Controlled ϕ Response for Multiple Input Control Case 1

TABLE 3.3 - GAIN FUNCTION FOR MULTIPLE CONTROL CASE 2

$$K_{ij}(\tau) = \sum_{n=1}^{10} A_n \cos(2(n-1)\pi\tau/T) + \sum_{m=1}^{10} B_m \sin(2m\pi\tau/T)$$

| | K_{11} | K_{12} | K_{21} | K_{22} |
|---------------|-------------|-------------|-------------|-------------|
| cosine terms; | | | | |
| A_1 | 0.2403 E-6 | 0.1500 E-6 | 0.4410 E-7 | 0.4434 E-8 |
| A_2 | -3.4555 | -1.5559 | -0.6102 | -3.3138 |
| A_3 | -0.1553 E-6 | -0.2150 E-7 | 0.1791 E-6 | 0.2293 E-7 |
| A_4 | 2.1350 | 3.6081 | 3.7095 | -1.2916 |
| A_5 | -0.1667 E-6 | -0.9843 E-7 | -0.2572 E-6 | -0.2303 E-6 |
| A_6 | 4.2537 | 4.2380 | 4.1580 | -1.2936 |
| A_7 | -0.2811 E-6 | -0.1318 E-6 | 0.9740 E-7 | 0.2793 E-8 |
| A_8 | -3.3500 | -0.6592 | 1.1674 | -1.5632 |
| A_9 | 0.4338 E-6 | 0.3102 E-6 | 0.6560 E-7 | -0.4223 E-9 |
| A_{10} | -5.6257 | -1.0427 | -1.3700 | -2.4263 |
| sine terms; | | | | |
| B_1 | -7.6279 | -3.4714 | 0.7585 E-1 | 0.6094 |
| B_2 | 0.3208 E-6 | 0.2097 E-6 | 0.1804 E-6 | 0.1018 E-6 |
| B_3 | -0.3559 | -0.4705 | -4.5742 | -2.9586 |
| B_4 | 0.6108 E-7 | 0.4998 E-7 | 0.3896 E-7 | 0.2578 E-7 |
| B_5 | -0.8576 | 0.7912 | 5.6375 | 2.1437 |
| B_6 | -0.2935 E-6 | -0.2241 E-6 | -0.1689 E-6 | -0.1106 E-6 |
| B_7 | 8.8409 | 3.4427 | -0.3819 E-3 | -0.4319 |
| B_8 | -0.1422 E-6 | -0.6702 E-7 | -0.7368 E-7 | -0.5076 E-7 |
| B_9 | -6.2437 | -1.7123 | 1.8233 | -0.3279 |
| B_{10} | 0.2806 E-6 | 0.1782 E-6 | 0.1487 E-6 | 0.1030 E-6 |

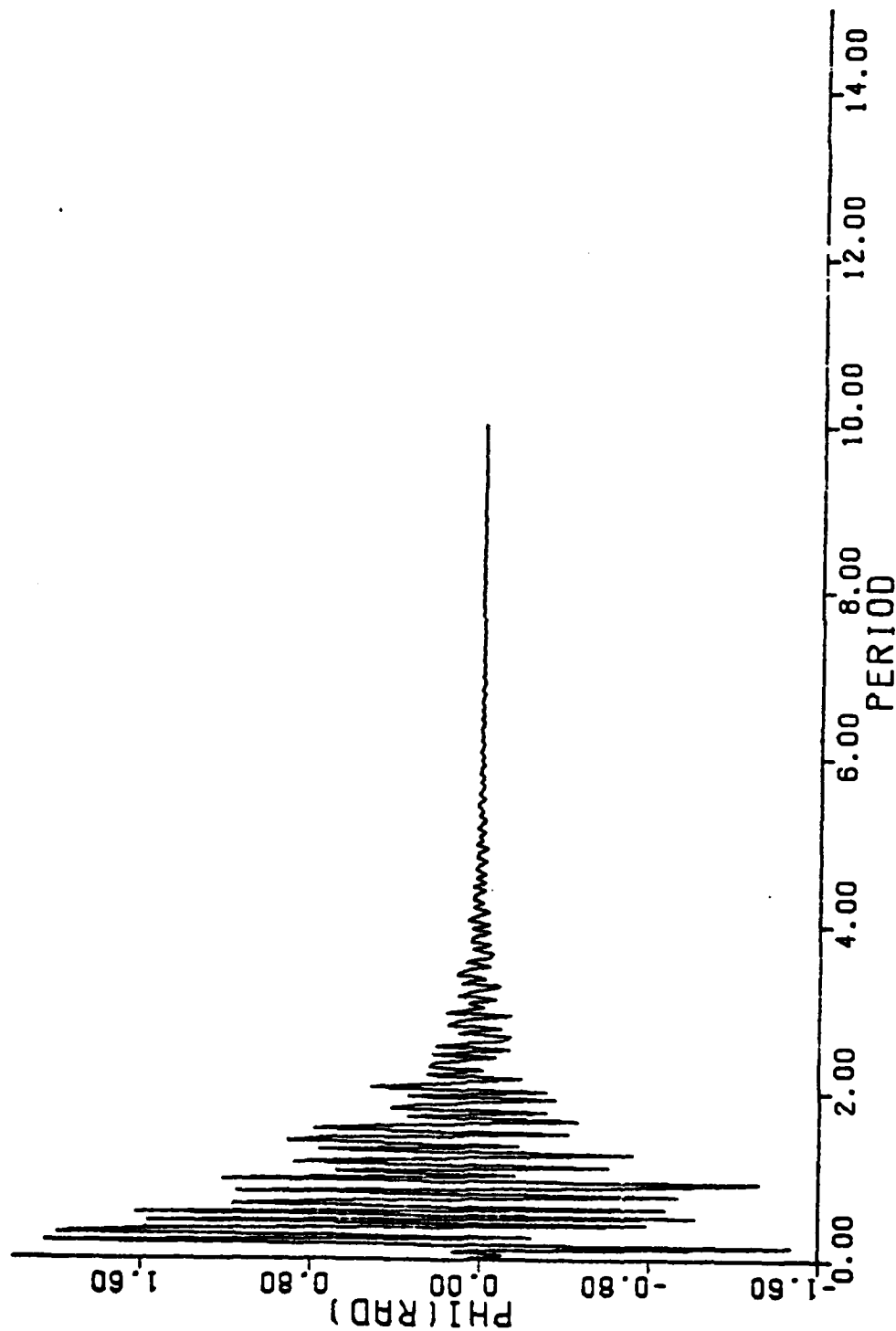


FIGURE 3.20 - Controlled ϕ Response for Multiple Input Control Case 2

CHAPTER 4

CONCLUSIONS

There are some important conclusions which can be drawn from the results presented in the previous chapter. Foremost is the contrast between the scaler control and the multiple input control. The use of these two control schemes in simultaneously moving two unstable characteristic exponents was demonstrated in the results section. The problem of moving two roots simultaneously is of particular importance to satellite designers, due to the number of cases where two, or more, unstable modes exist. In this study the case where there are two unstable modes was shown to outnumber the cases of one unstable mode. It would be overly restrictive to limit satellite designs to only those cases of zero or one unstable mode. In cases with multiple unstable modes a scaler control may be applied repetitiously to control a single mode at a time. This involves applying a scaler control to the original system to take care of one mode, then forming a new "semicontrolled" system to which another scaler control is applied. This process is continued until all modes are controlled as desired. The problem here is that the numerical difficulties quickly make this scheme unworkable.

The primary emphasis of this study has been to develop a workable scheme to control a periodic system with two unstable modes. From the results presented it is obvious the two types of controllers offer significant differences in achieving

this control. The scaler control works with simple workable feedback gains, but does not offer predictable results. The multiple input control system offers predictable results as to the placement of the new stable characteristic exponents, but requires complicated gains which are time dependent. The greater the accuracy desired in the placement of the characteristic exponents, the more complicated the gain time functions become. These complicated gains are unavoidable in the multiple input controller. Therefore the problem becomes, how to predict the placement of the characteristic exponents using the scaler control.

The only clue in predicting the results of the scaler controller comes from the relation between the characteristic exponent's sum and the trace of the matrix defining the system. Through the use of the theorem given by equation (2.86) a locus of controlled characteristic exponents was plotted. From this plot there can be found a breakin point, unique for a given gain ratio. If this breakin point can be predicted the problem will be effectively solved. Also from these plots it is obvious that some optimum ratio exists that gives the most negative breakin point for the system. As was discussed the breakin point can be thought of as the most stable controlled point. More negative values of one characteristic exponent can be achieved, but it requires the second characteristic exponent to be less negative. Hence the breakin point for the optimum ratio represents the most desirable location of the controlled characteristic exponents. As of yet there is no clear way to predict breakin points or even the optimum.

ratio.

If a less specific solution is desired, that is one that is simply stable and not necessarily the most stable, then the imaginary axis crossings must be predicted. For cases similar to the one presented, (that is one with negative breakin points), the first imaginary crossing, as the gain is increased from zero, occurs when the sum computed from the trace theorem, is equal to zero. The second crossover occurs as one branch heads toward the origin after breaking into the real axis. This crossover was difficult to find using computer numerical integration. Whether a crossover actually occurs at the origin is still unclear. These two crossover points define the region where the system is guaranteed stable.

Bibliography

- 1) Boyce, William E., and DiPrima, Richard C., Elementary Differential Equations and Boundary Value Problems, John Wiley & Sons, New York, 1977.
- 2) Calico, R.A., & Yeakel, G.S., "Active Attitude Control of a Spinning Symmetric Satellite in an Elliptical Orbit", Air Force Institute of Technology, 1982.
- 3) Jordan, D.W., and Smith, P., Nonlinear Ordinary Differential Equations, Clarendon Press, Oxford, 1979.
- 4) Kane, T.R., and Barba, P.M., "Attitude Stability of a Spinning Satellite in an Elliptic Orbit", Journal of Applied Mechanics, pp.402-405, June 1966.
- 5) Kane, T.R., and Shippy, D.J., "Attitude Stability of a Spinning Unsymmetric Satellite in a Circular Orbit", The Journal of the Astronautical Sciences, Vol. X, No. 4, pp.114-119, Winter 1963.
- 6) Kaplan, Marshall H., Modern Spacecraft Dynamics & Control, John Wiley & Sons, New York, 1976.
- 7) Meirovitch, Leonard, Methods of Analytical Dynamics, McGraw-Hill Book Company, New York, 1970.
- 8) Shelton, W.L., Modal Control of a Satellite in Orbit about L3, M.S. Thesis, Wright-Patterson AFB, Ohio: Air Force Institute of Technology, Dec. 1980.
- 9) Wiesel, W., Shelton, W., "Modal Control of an Unstable Periodic Orbit", AAS/AIAA Astrodynamics Specialist Conference, Paper 81-146, Lake Tahoe, Nevada, August 1981.

- 10) Vinh, Nguyen X., Busemann, Adolf, Culp, Robert D., Hypersonic and Planetary Entry Flight Mechanics, The University of Michigan Press, Ann Arbor, 1980.
- 11) Yeakel, G.S., Active Control of Linear Periodic Systems, M.S. Thesis, Wright-Patterson AFB, Ohio: Air Force Institute of Technology, Dec. 1981.

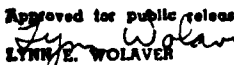
Vita

Gregory E. Myers was born 1 January 1960, in Harrisburg Pennsylvania. He was raised in New Bloomfield, Pennsylvania and graduated from Wesley Perry High School in 1977. He attended the University of Michigan with a scholarship from AFROTC. In 1981 Myers graduated from Michigan with a Bachelor of Science in Engineering, in Aerospace engineering. Upon graduation he accepted a Reserve Commission from the Air Force and was accepted at the Air Force Institute of Technology, Wright-Patterson AFB Ohio.

G. Myers worked on his Master of Science in Aeronautical Engineering while at AFIT. Upon graduation in December 1982 he will be reassigned to the 6595th Shuttle Test Group, Vandenberg AFB California.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|-------------------------------------|--|
| 1. REPORT NUMBER AFIT/GAE/AA/82D-21 | 2. GOVT ACCESSION NO. AD A124698 | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle) ACTIVE CONTROL OF LINEAR PERIODIC SYSTEM WITH TWO UNSTABLE MODES | | 5. TYPE OF REPORT & PERIOD COVERED MS Thesis |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s) Gregory E. Myers, 2nd Lt. USAF | | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS AIR FORCE INSTITUTE OF TECHNOLOGY AFIT/ENA WRIGHT-PATTERSON AFB, OH 45433 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS | | 12. REPORT DATE December 1982 |
| | | 13. NUMBER OF PAGES 78 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) | | 15. SECURITY CLASS. (of this report) Unclassified |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |
| 16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited | | |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) | | |
| 18. SUPPLEMENTARY NOTES <p>Approved for public release: IAW AFR 100-17.  LYNN E. WOLAVER Dean for Research and Professional Development Air Force Institute of Technology (ATC) Wright-Patterson AFB OH 45433</p> 19 JAN 1983 | | |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Active Control Characteristic Exponents Floquet Theory Modal Control Periodic Matrix Satellite Attitude | | |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The attitude of a spinning symmetric satellite in an elliptical orbit was analyzed. The linearized equations were formed from which the stability was determined using Floquet theory. The majority of satellite configurations examined exhibited two unstable modes. Control laws using pole placement techniques were implemented which stabilize the satellite. A scalar control provided stability but did not allow for exact pole placement of the controlled system. Multiple input control gave exact pole placement but required complicated time dependent gains. | | |

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)